The Effect of Wind Shear on the Performance of a Horizontal Axis Wind Turbine

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ABSTRACT

This paper describes a method for calculating the effect of wind shear on the turbine mean output, periodic variations in torque, pitching moments and yawing moments. The numerical results obtained with the present method agree well with experimental and other numerical data.

INTRODUCTION

A wind shear arises from the fact that the wind near the surface of the earth is not entirely uniform. There exists a large scale boundary layer which may cause significant load variations on a blade as it passes from the bottom to the top of a rotation. The rotor of a wind turbine is placed at some distance above the ground to avoid the extreme low velocity part of the atmospheric boundary layer. The influence of wind shear on the power output and the blade loading of a horizontal axis wind turbine is complicated, because each blade element is subjected to a varying wind velocity during a revolution of the rotor. The forces and moments induced by vertical wind gradient are illustrated in Fig. 1. The cyclic loads which arise from the wind shear may create serious resonances in the blade or supporting structure.

A modified strip theory approach has been used to determine the effects of wind gradient. This model takes into account coning, tilting, azimuth and non-axial flows.

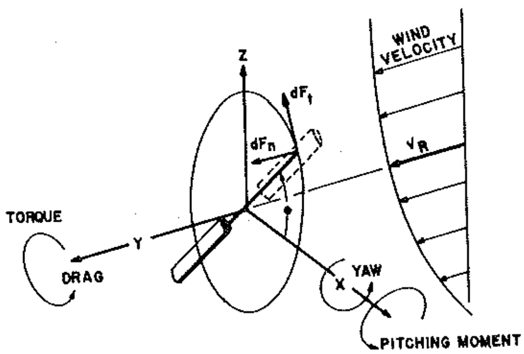

Fig. 1 Different forces and moments acting on a rotor[5].

VELOCITY COMPONENTS AT THE BLADE

For calculating the forces and moment on a blade, the velocity components of the air flow relative to any point on the blade and also the induced velocity components have to be known. Several reference frames are considered to include the effect of wind shift, tilt, azimuth and coning. This has been discussed in Appendix A.

The variation of wind velocity with height can be presented analytically by a power law which is expressed as

$$\frac{V}{V_{ref}} = \left(\frac{Z}{Z_{ref}}\right)^{\eta} \tag{1}$$

Radial distance of a particular point on the blade can be expressed in S_o co-ordinate system as

$$\begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} = [K_T]\ [K_\theta]\ [K_B] \begin{vmatrix} O \\ O \\ R_L \end{vmatrix} \tag{2}$$

where R_L is the distance of a point from the root of the blade.

Equation (2) can be expressed as

$$\begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} = \begin{vmatrix} R_L \cos\beta \sin\theta_k \\ -R_L \sin\beta \cos\alpha_T - R_L \cos\beta \cos\theta_k \sin\alpha_T \\ -R_L \sin\beta \sin\alpha_T + R_L \cos\beta \cos\theta_k \cos\alpha_T \end{vmatrix} \tag{3}$$

The height of a particular point of the blade from the ground level can be written as

$$Z = Z_{ref} + Z_o$$

or,

$$Z = Z_{ref} + R_L\ (-\sin\beta \sin\alpha_T + \cos\beta \cos\theta_k \cos\alpha_T) \tag{4}$$

Considering Z_{ref} as the hub height of a wind turbine from the ground level and putting the value of Z in equation (1), one finds

$$V_{\alpha_o} = V_{ref}\,[(H + R_L\ (-\sin\beta \sin\alpha_T + \cos\beta \cos\theta_k \cos\alpha_T))/H]^{\eta} \tag{5}$$

The velocity components acting on a blade element rotating at a radius r are shown in Fig. 2.

Now introducing the induced velocity in S_3 co-ordinate system as

$$\bar{V}_{s_3} = \begin{vmatrix} V_t \\ V_a \cos\beta \cos\alpha_T \\ V_\nu \end{vmatrix} \tag{6}$$

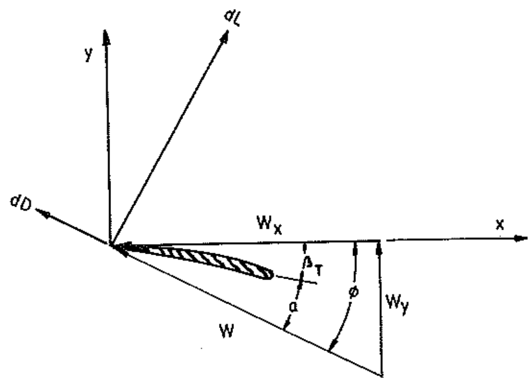

Fig. 2 Velocity diagram for a rotor blade element.

Equation (6) can be written as

$$\bar{V}_{s_3} = \begin{vmatrix} \Omega\, r \cos\beta\, a' \\ V_{\alpha}\, a \cos\beta \cos\alpha_T \sin\gamma \\ V_v \end{vmatrix} \tag{7}$$

The rotational motion of the blade will add a velocity component $\Omega\, r \cos\beta$ in the total velocity vector relative to the blade and this component can be written in S_3 co-ordinate system as

$$V_{b_1} = \begin{vmatrix} \Omega\, r \cos\beta \\ 0 \\ 0 \end{vmatrix} \tag{8}$$

The components of relative velocity W can be expressed as

$$W_x = V_{\alpha_o} \cos\gamma \cos\theta_k + V_{\alpha_o} \sin\gamma \sin\theta_k \sin\alpha_T - \Omega\, r \cos\beta\, (1 + a') \tag{9}$$

$$W_y = V_{\alpha_o} [\sin\gamma \cos\alpha_T \cos\beta\, (1-a) + \sin\beta \sin\theta_k \cos\gamma - \sin\gamma \sin\beta \sin\alpha_T \cos\theta_k] \tag{10}$$

The local angle of attack α is defined as

$$\alpha = \phi - \beta_T = tan^{-1} \frac{W_y}{W_x} - \beta_T \tag{11}$$

AERODYNAMIC FORCES

To calculate the interference factors a and a', the expression for the torque and thrust from momentum theory and blade element theory are equated. From momentum theory the torque and thrust on a radial element can be written as

$$dT = (2\rho r \cos^2\beta \cos^2\alpha_T \sin^2\gamma\, V_{\alpha_o}^2)\, (a(1-a))\, dr\, d\theta\, F \tag{12}$$

$$dQ = (2\rho r^3\ a'\ (1-a)\ V_{\alpha_o}\ \cos\alpha_T \sin\gamma \cos^4\beta)\ \Omega\ dr\ d\theta\ F \tag{13}$$

Here F is the tip loss factor defined as

$$F = \frac{B\Gamma}{\Gamma_\alpha}$$

where Γ = circulation at a radial station r

Γ_α = corresponding circulation for a rotor with infinite number of blades.

The tip loss factor has been derived using the method of Prandtl[1]. From blade element theory

$$dT = \tfrac{1}{2}\rho\ W^2\ \cos\phi\ (C_L + C_D \tan\phi)\ \frac{BC\cos\beta\ dr\ d\theta}{2\pi} \tag{14}$$

$$dQ = \tfrac{1}{2}\rho\ W^2\ \sin\phi\ (C_L - C_D \frac{1}{\tan\phi})\ \frac{BC\,r\cos\beta\ dr\ d\theta}{2\pi} \tag{15}$$

where

$$W^2 = W_x^2 + W_y^2 \tag{16}$$

By equating both sets of equation for thrust and torque the induced velocity factors a and a' may be determined.

$$a(1-aF) = \frac{1}{8}\frac{\sigma\ W^2\ \cos\phi\ C_L}{\cos\beta\ \cos^2\alpha_T\ \sin^2\gamma\ V_{\alpha_o}^2\ F} \tag{17}$$

$$a'(1-aF) = \frac{1}{8}\frac{\sigma\ W^2\ \sin\phi\ C_L}{r\ \cos\alpha_T\ \sin\gamma\ \cos^3\beta\ V_{\alpha_o}\ F\ \Omega} \tag{18}$$

The drag terms have been omitted in equations (17) and (18), on the basis that the retarded air due to drag is confined to thin helical sheets in the wake and have little effect on the induced flows[2].

The elemental thrust and torque coefficients can be expressed by the following equations:

$$dC_T = \frac{8}{\pi R^2}\ (V_{\alpha_o}/V_\alpha)^2\ aF\ (1-aF)\ \cos^2_\beta\ \cos^2\alpha_T\ \sin^2_\gamma\ (1 + \frac{C_D}{C_L}\tan\phi)\ dr\ d\theta\ r \tag{19}$$

$$dC_Q = \frac{8}{\pi R^3}\ \frac{V_{\alpha_o} r^3}{V_\alpha^2}\ a'F\ (1-aF)\ \cos\alpha_T\ \sin\gamma\ \cos^4\beta\ (1 - \frac{C_D}{C_L}\frac{1}{\tan\phi})\ \Omega\ dr\ d\theta \tag{20}$$

RESULTS AND DISCUSSIONS

Results are presented for a two-blade 47 m wind turbine with a tip speed ratio 8, constant rpm, variable pitch and a coning angle of 5 degrees. The results are calculated for a wind power law exponent of 1/6 and for the present analysis the hub height is considered as the reference height. Figures 3 and 4 show the variation of power coefficient and thrust coefficient with azimuth. Figures 5 and 6 show the variation of pitching moment and yawing moment with azimuth. The wind shear produces a periodic wind load with a frequency of twice per revolution. A large variation of pitching moment and yawing moment will occur when one blade is at 180° position and the other blade at 0° position. In this condition the upper blade will receive maximum wind velocity and the lower blade will receive minimum wind velocity. Figures 7 and 8 show the loads exerted on the tower by the rotor. As expected the resultant axial force will be maximum when the blades are in the horizontal position. But the total tangential and the vertical force will be zero due to the opposite direction of forces coming from the two at that position. The variation of forces on the tower top will be twice per revolution due to the two-blade configuration.

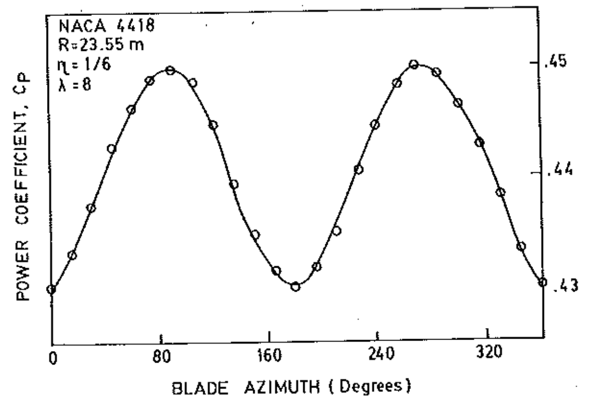

Fig. 3 Effect of wind shear on power coefficient.

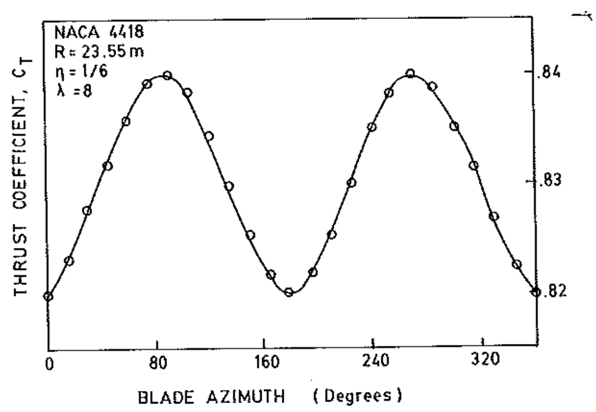

Fig. 4 Variation of thrust coefficient with azimuth due to wind shear.

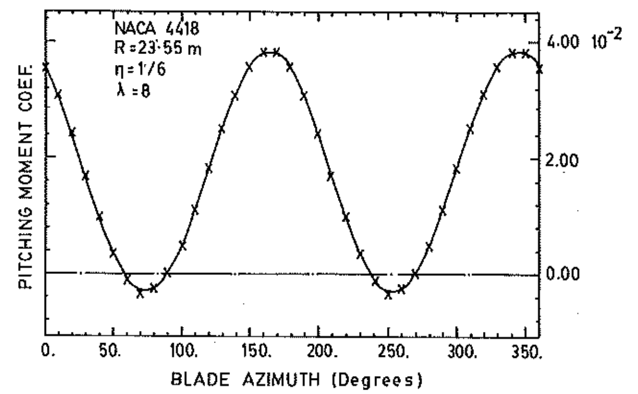

Fig. 5 Effect of wind shear on pitching moment.

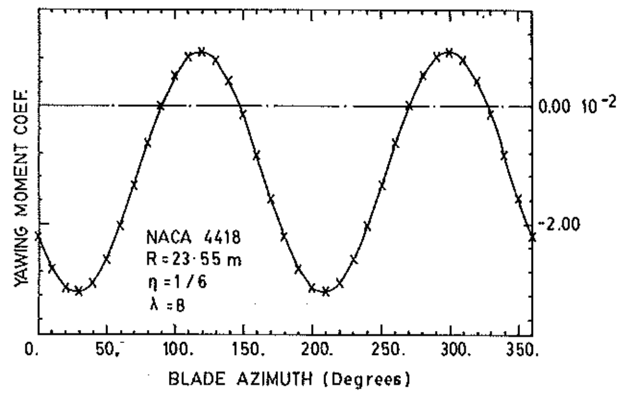

Fig. 6 Effect of wind shear on yawing moment.

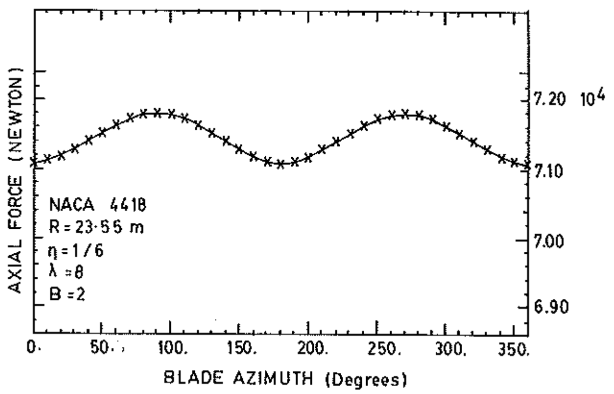

Fig. 7 Variation of tower top axial force due to wind shear during one revolution.

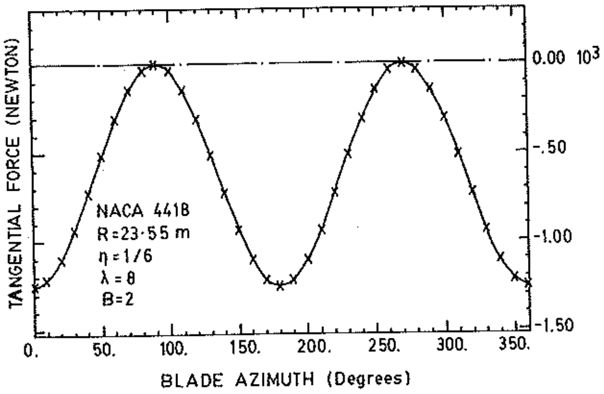

Fig. 8 Variation of tower top tangential force due to wind shear during one revolution.

Figure 9 shows the effect of wind shear on flapwise bending moment of the Vestas 15 wind turbine. The Vestas 15 is a 55 kW stall regulated propeller type rotor using NACA 4416-4424 profile series for its blade. The diameter of the three-blade rotor is 15.34 m. For a blade in the upper half of the rotor disk the axial velocity will be higher than for a blade in the lower half of the disk. The increase in wind velocity increases both the resultant velocity and the angle of attack α. The variation in angle of attack in turn will cause variations in the normal and tangential forces with the angle of rotation. Experimental data of Fig. 9 is taken from reference (3). The lowered values measured may be due to the reduction of axial force by the centrifugal force. The results predicted by the present method are closer to the experimental values than the predicted values of reference (3).

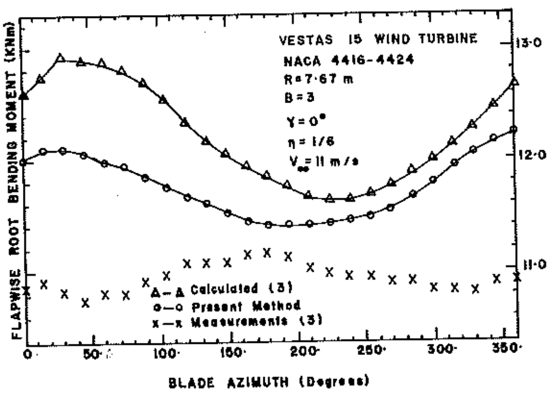

Fig. 9 Comparison of measured and calculated blade flapwise root bending moment due to wind shear.

Figure 10 illustrates the percentage decrease in turbine output due to wind gradient for the Smith-Putnam wind turbine. The Smith-Putnam wind turbine blade employed NACA 4418 airfoil which has a continuous twist of 5° along a 19.82 meter length. The rated power of this two-blade constant chord turbine is 1.25 MW. The variation of power due to the wind gradient is

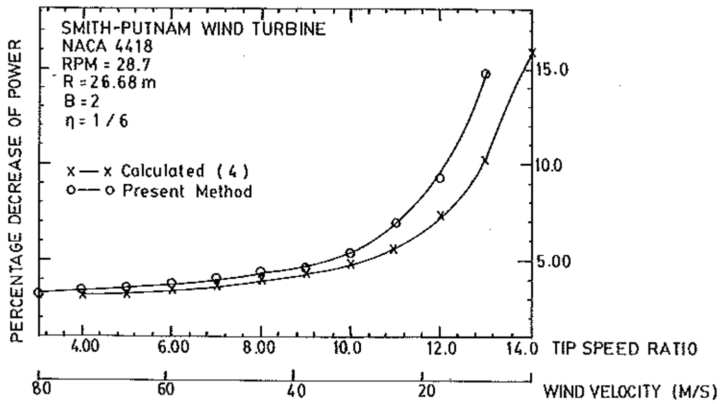

Fig. 10 Percentage reduction in power output due to wind gradient of Smith-Putnam wind turbine.

approximately constant over a wide range of tip speed ratios for this turbine. The effect of wind shear is much more pronounced at low speeds. However, the effect for fatigue stresses is negligible at these low speeds. The percentage variation in turbine output changes appreciably as the net output approaches zero.

CONCLUSIONS

For a two-blade horizontal axis wind turbine operating with uniform velocity and without any disturbances the loads will be steady. The introduction of wind shear will cause each blade to experience a periodic force at a frequency of once per revolution and result in a yawing and pitching moment at a frequency of twice per revolution for the whole turbine.

For the design of a horizontal axis wind turbine the combined influence of wind gradient, wind shift, coning, tilting and blade azimuth should be considered. Presently, all manufacturers of large wind turbines include the wind shear in their designs.

NOMENCLATURE

a	axial interference factor
a'	tangential interference factor
B	number of blades
C	Blade chord
C_p	power coefficient, $P/\frac{1}{2}\rho A\ V_\alpha^3$
C_L	lift coefficient
C_D	drag coefficient
C_{mp}	pitching moment coefficient, $M_p/\frac{1}{2}\rho A\ V_\alpha^2 R$
C_{mz}	yawing moment coefficient, $M_{yaw}/\frac{1}{2}\rho V_\alpha^2 AR$

C_T	thrust coefficient, $T/\frac{1}{2}\rho A\, V_\infty^2$
F	tip loss factor
M_p	pitching moment
M_{yaw}	yawing moment
P	power
r	local radius
R	rotor
T	thrust
V_{ref}	reference wind velocity
V_∞	undisturbed wind velocity
W	relative wind velocity
Z_{ref}	height of reference point
α	angle of attack
α_T	tilt angle
β	coning angle
β_T	twist angle
γ	yawing angle
θ_k	blade azimuth angle
λ	tip speed ratio
λ_r	local tip speed ratio
V_a	induced axial velocity
V_t	induced tangential velocity
V_v	induced vertical velocity
ϕ	angle of relative wind velocity
ρ	air density
Ω	angular speed of rotor
σ	solidity
η	power law exponent

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APPENDIX A: Local Reference Frames

To calculate the aerodynamic forces acting on the rotor several co-ordinate systems are introduced in the present analysis. These frames include a reference frame S_o fixed at the top of the tower of the wind turbine with Z_o being the vertical axis and (X_o, Y_o) forming the horizontal plane. A second non-rotating frame S_1 fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance and a rotation of tilting angle α_T around the X_o axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally, a local reference frame S_3 is attached to a particular point of the blade at a distance r from the hub and is rotated over a coning angle β. The relationships between the reference frames can be expressed as

$$S_1 = [K_T]\, S_o, \quad S_2 = [K_\theta]\, S_1, \quad S_3 = [K_\beta]\, S_2$$

The transformation matrices are:

$$\text{for tilting angle, } K_T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_T & -\sin\alpha_T \\ 0 & \sin\alpha_T & \cos\alpha_T \end{vmatrix}$$

$$\text{for azimuth angle, } K_\theta = \begin{vmatrix} \cos\theta_k & 0 & \sin\theta_k \\ 0 & 1 & 0 \\ -\sin\theta_k & 0 & \cos\theta_k \end{vmatrix}$$

$$\text{for coning angle, } K_\beta = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{vmatrix}$$