Optimum Tilt Angle Calculations for South Facing Solar Collectors at Karachi (Pakistan)

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ABSTRACT

The values of tilt factors for the conversion of total solar radiation on a horizontal surface to that on a tilted surface facing south have been calculated for Karachi (latitude: $24^{\circ}.54'$ N). Tilt factors are given for six slopes, the latitude of the place ϕ, $\phi \pm 10^{\circ}$, $\phi \pm 20^{\circ}$ and for 60°. It has been found that an optimum insolation of solar radiation during the heating season (September-March) is obtained for $\phi + 20^{\circ}$ tilt, and for the cooling season (April-August) a horizontal surface and a tilt of $\phi - 20^{\circ}$ is favourable.

INTRODUCTION

Most solar radiation data is available for horizontal surfaces. It is therefore desirable to estimate the effect of orientation of the receiving surface so as to optimize the total solar radiation falling on it. This approach has been successfully employed for optimum utilization of flat-plate solar collectors for practical purposes.

Flat-plate solar collectors absorb both direct and diffuse components of solar radiation. The angular correction for direct radiation is quite simple, but for diffuse components it depends on different factors. The correction for diffuse radiations depends on their distribution over the sky. This distribution depends particularly on clouds, and also on spatial distribution and the amount of other atmospheric components that determine scattering. It is further assumed that the properties of the ground or other surfaces "seen" by a tilted surface reflects solar radiations in such a way as to be a source of diffuse radiation equivalent to the sky. The radiation on a tilted surface under these conditions is

$$H_T = H_b R_b + H_d, \tag{1}$$

where H_b and H_d are the direct and diffuse components of total solar radiation, H, on the horizontal surface and R_b is the beam correction factor, given by Hottel and Woertz as

$$R_b = \frac{\cos(\phi - s)\cos\delta\cos\omega_s + \sin(\phi - s)\sin\delta}{\cos\phi\cos\delta\cos\omega_s + \sin\phi\sin\delta} \tag{2}$$

In equation (2), ϕ is the lattitude of the place and s, is the tilt angle; whereas δ and ω_s are the solar declination and sunset hour angles respectively. It can be seen that R_b goes to unity for $s = 0$ (horizontal surface). The sunset hour angle can be determined as

$$\cos\ \omega_s = -\tan\phi\ \tan\delta \tag{3}$$

The solar declination δ, for the 15th of each month has been taken from Liu and Jordan (3) and is shown in Table 1.

Table 1
Solar declination δ
(for 15th of each month)

Months	δ	Sinδ	cosδ
January	–21.16′	–0.362	0.932
February	–12.56′	–0.223	0.974
March	– 2.26′	–0.042	0.999
April	9.30′	0.165	0.986
May	18.41′	0.320	0.947
June	23.17′	0.395	0.918
July	21.39′	0.368	0.929
August	14.18′	0.247	0.969
September	3.20′	0.058	0.998
October	– 8.14′	–0.143	0.989
November	–18.18′	–0.314	0.949
December	–23.14′	–0.394	0.918

(Reference: 3)

The effective ratio, R, of solar energy on a tilted surface to that on a horizontal surface is then

$$R = H_T/H = H_b/H + H_d/H \tag{4}$$

Liu and Jordan (4) have improved this model by considering the radiation on a tilted surface to be made up of three components: the direct radiations, the diffuse radiations and the solar radiations reflected from the ground, which the tilted surface "sees". The tilted surface also sees ground or other surroundings, and has a reflectance, ρ, for solar radiations.

According to Liu and Jordan (ibid), the total solar radiation on a tilted surface at any time is

$$H_T = H_b\, R_b + H_d \frac{1 + \cos s}{2} + (H_b + H_d) \frac{(1 - \cos s)\, \rho}{2} \tag{5}$$

and by the definition of R,

$$R = \frac{H_b}{H} R_b + \frac{H_d}{H} \frac{(1 + \cos s)}{2} + \frac{(1 - \cos s)\ \rho}{2} \tag{6}$$

where s is the tilt angle and ρ the ground reflectance. Liu and Jordan have suggested that the value of ground reflectance is 0.2 when there is no snow, and 0.7 when there is snow cover.

METHOD AND CALCULATIONS

In order to work out the ratio, R, to convert the total solar radiations on a horizontal surface to that for a tilted surface, a knowledge of H, the total solar radiation, is required. The value of the total solar radiation, H, for Karachi on a horizontal surface for different months were taken from Firoz Ahmad et al. (5) while the ratios of direct to total (H_b/H) and diffuse to total (H_d/H) are from the results of Firoz Ahmad et al. (6), and are given in Table 2.

Table 2
Percentage variation of direct and diffuse solar radiations at Karachi (Pakistan)

Months	% Direct radiations	% Diffuse radiations
January	79	21
February	75	25
March	73	27
April	67	33
May	62	38
June	59	41
July	50	50
August	55	45
September	65	35
October	75	25
November	81	19
December	83	17

(Reference: 6)

Firstly R_b was calculated for January to December inclusive for various tilt angles ($s = \phi$ $s = \phi \pm 10^\circ$, $s = \phi \pm 20^\circ$, and $s = 60^\circ$) using the relationship given in equation (2). After computing R_b for various tilt angles and months, the conversion factors, R, was calculated using equation (6). The value of ρ, the ground reflectance, for Karachi was assumed to be 0.2 for these calculations.

RESULTS

The value of the total solar radiations received for various tilts of the solar collectors are given in Table 3, whereas the conversion factor, R, calculated from equation (6) for various tilt angles is tabulated in Table 4.

The total annual collection of solar radiations in MJ/m^2 and its seasonwise breakup is compiled in Table 5 (a), (b) and (c). The analysis of the data has been so planned as to get an idea for suitable use and for prediction of efficient performance of solar collectors at Karachi. According

Table 3
Solar radiations in MJ/m² per day for various tilt angles of solar collectors at Karachi ($\phi = 24°.54'$ N)

Months	Angles						
	$s = 0$	$s = \phi - 20$	$s = \phi - 10$	$s = \phi$	$s = \phi + 10$	$s = \phi + 20$	$s = 60$
January	17.2	18.5	21.0	22.9	24.3	25.1	24.9
Febuary	19.4	20.4	21.9	23.3	23.8	23.9	22.8
March	21.4	22.0	22.9	23.3	23.1	22.4	20.1
April	21.3	21.4	21.4	21.2	20.3	19.0	18.7
May	20.4	20.3	20.1	19.3	18.0	16.5	13.5
June	18.7	18.5	18.0	17.2	15.9	14.4	11.6
July	14.7	14.6	14.3	13.6	12.8	11.4	9.7
August	15.8	15.7	15.7	15.2	15.3	13.4	11.4
September	18.5	18.8	19.2	19.2	18.7	17.8	15.7
October	19.9	20.7	22.1	22.9	23.1	22.8	21.3
November	19.8	21.2	23.6	25.7	26.7	27.5	26.9
December	17.4	19.0	21.9	24.2	25.8	27.0	27.1

Table 4
Values of the conversion factor R, for various tilt angles at Karachi (Pakistan) $\phi = 24°.54'$ N

Months	Angles						
	$s = 0$	$s = \phi - 20$	$s = \phi - 10$	$s = \phi$	$s = \phi + 10$	$s = \phi + 20$	$s = 60$
January	1.0	1.079	1.220	1.330	1.414	1.457	1.447
February	1.0	1.051	1.126	1.200	1.225	1.232	1.176
March	1.0	1.029	1.070	1.090	1.081	1.048	0.951
April	1.0	1.006	1.009	0.990	0.955	0.984	0.868
May	1.0	0.997	0.981	0.940	0.883	0.806	0.661
June	1.0	0.993	0.966	0.920	0.853	0.768	0.619
July	1.0	0.996	0.975	0.930	0.874	0.780	0.660
August	1.0	0.999	0.994	0.960	0.967	0.850	0.720
September	1.0	1.017	1.036	1.030	1.009	0.961	0.850
October	1.0	1.041	1.110	1.140	1.160	1.146	1.070
November	1.0	1.071	1.194	1.290	1.351	1.389	1.358
December	1.0	1.090	1.257	1.390	1.480	1.550	1.555

Table 5
Solar radiation data for horizontal and tilted surface at Karachi (Pakistan) lattitude: 24°.54′ N

Tilt angles	Average solar radiations MJ/m^2 per day	Annual total solar radiations GJ/m^2
s = 0	18.7	6.830
s = ϕ – 20	19.3	7.028
s = ϕ – 10	20.1	7.349
s = ϕ	20.6	7.532
s = ϕ + 10	20.6	7.532
s = ϕ + 20	20.1	7.334
s = 60	18.6	6.799

Heating season (September to March)

Tilt angles	Average solar radiations MJ/m^2 per day	Total solar radiations GJ/m^2
s = 0	19.1	4.047
s = ϕ – 20	20.1	4.260
s = ϕ – 10	21.8	4.615
s = ϕ	23.1	4.890
s = ϕ + 10	23.7	5.014
s = ϕ + 20	23.8	5.041
s = 60	22.7	4.819

Cooling season (April to August)

Tilt angles	Average solar radiations MJ/m^2 per day	Total solar radiations GJ/m^2
s = 0	18.1	2.778
s = ϕ – 20	18.1	2.778
s = ϕ – 10	17.9	2.739
s = ϕ	17.2	2.643
s = ϕ + 10	16.5	2.521
s = ϕ + 20	14.9	2.284
s = 60	12.9	1.969

to the calculations, the tilt angle for the best performance of the solar collector for round-the-year utilization is equal to the lattitude of the place. Fig. 1 shows the solar insolation for various tilt angles throughout the year.

For the purpose of optimum utilization of solar collectors, we classify the year into two sea-

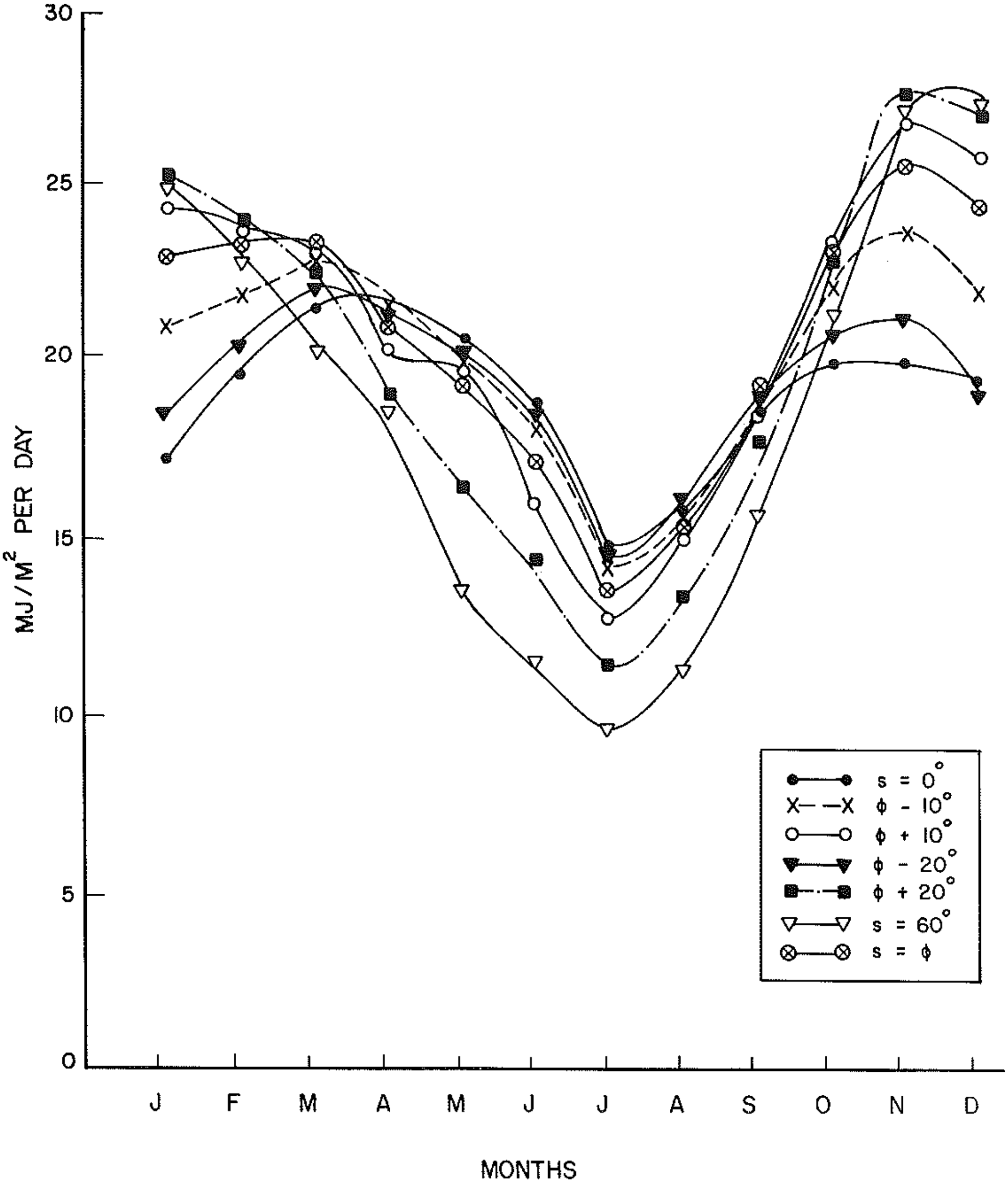

Fig. 1 Solar insolation for various tilt angles throughout the year for Karachi (Pakistan) Lattitude: 24°.54'N.

sons, namely the heating season and the cooling season. These seasons extend from September to March and from April to August respectively.

A tilt angle of $\phi + 20^\circ$ has been found to be the best for collecting solar radiations during the heating season whereas an angle of $\phi - 20^\circ$ has been found to give maximum solar insolation during the season in which solar radiations are required for cooling purposes (April-August). Fig. 2 shows a plot of solar insolation for various tilts of the solar collectors during the heating

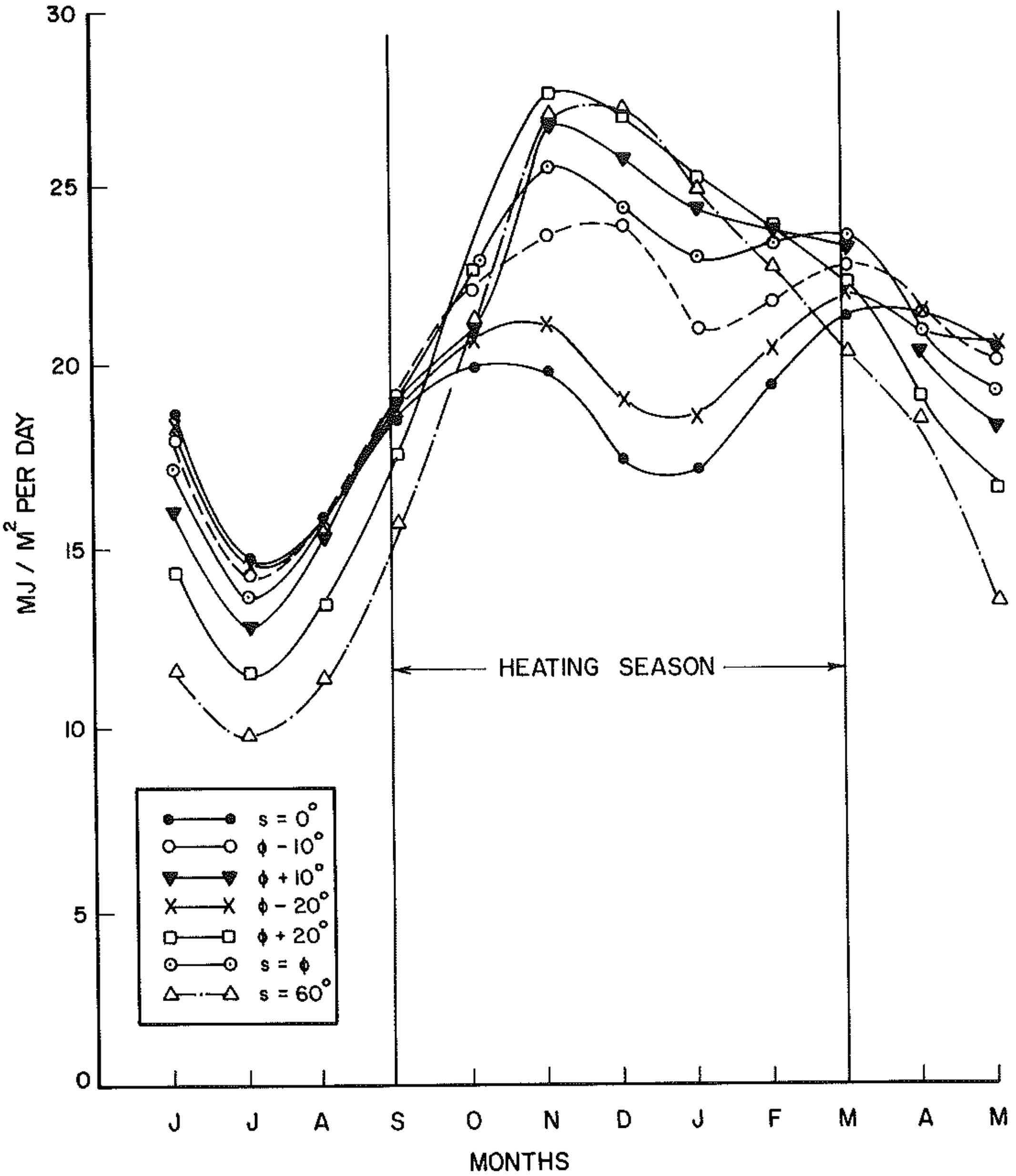

Fig. 2 A plot of solar insolation for various tilts of the solar collectors during heating season.

season. Fig. 3 shows the ratio R (H_T/H) for the heating and cooling seasons, which shows a pronounced maxima for the winter months (September to March). Average daily solar insolation from January to December for each tilt angle is shown in Fig. 4. This shows the effect of orientiation on the value of solar radiations.

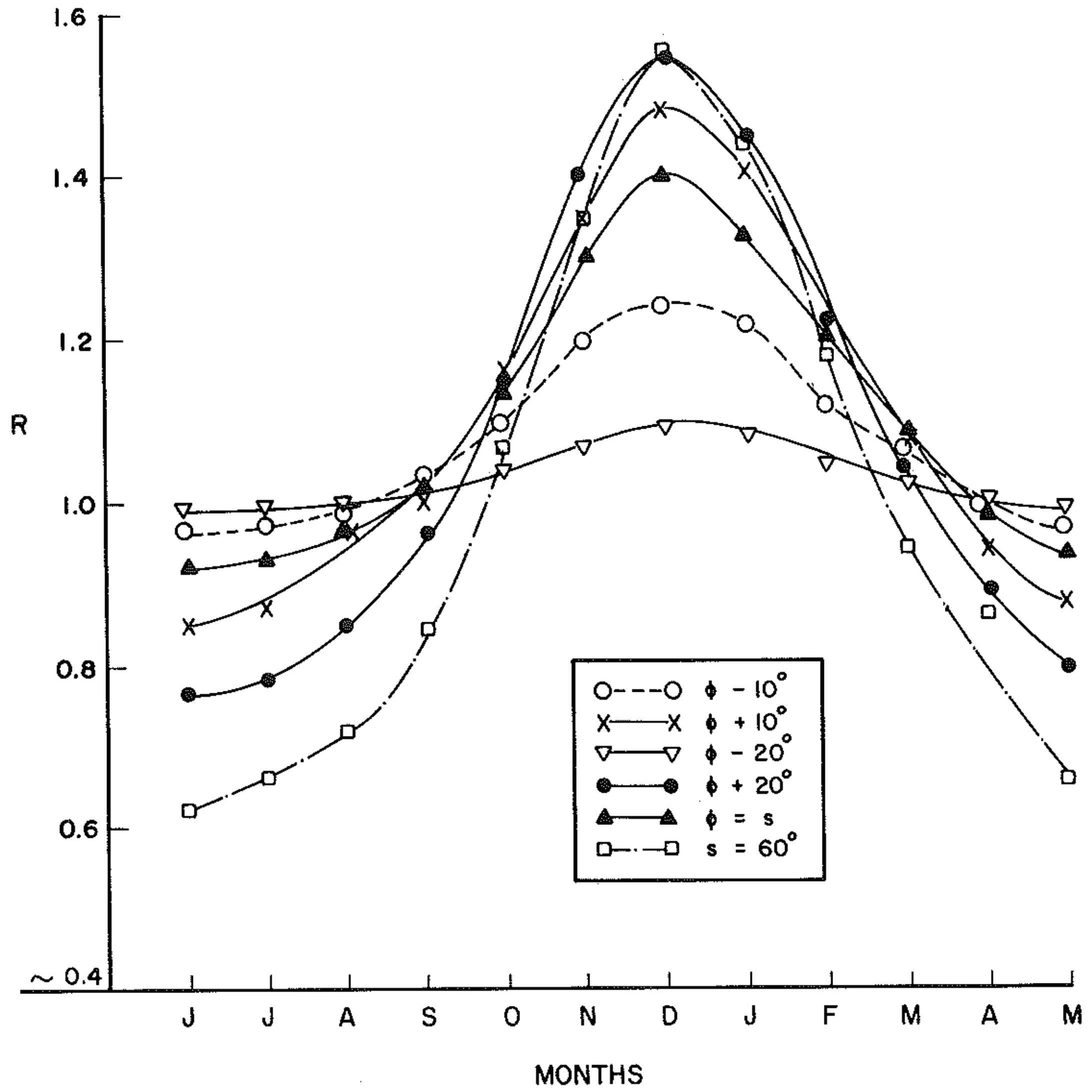

Fig. 3 Ratio $R = H_T/H$, the conversion factor for various tilt angles of solar collectors at Karachi (Pakistan)

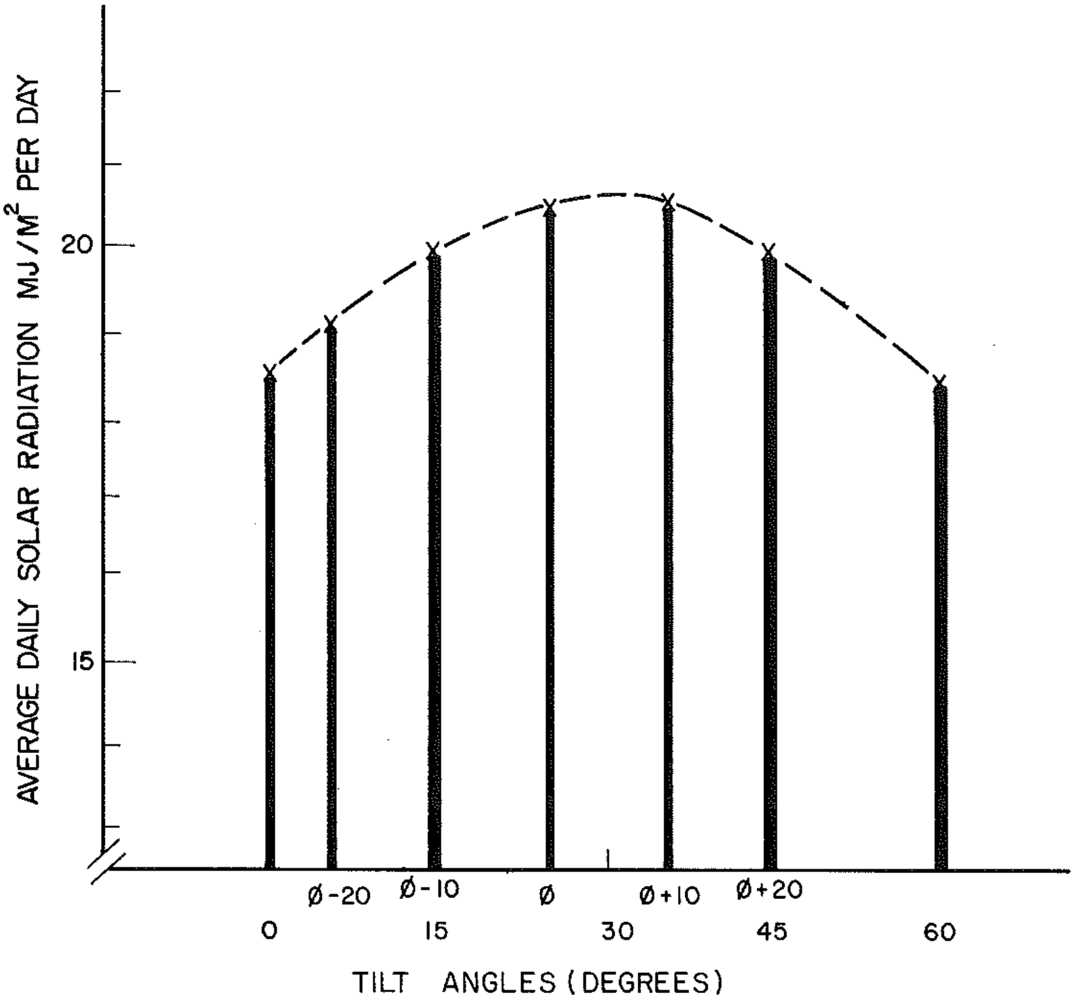

Fig. 4 Average solar radiations for various tilt angles at Karachi (Pakistan) lattitude: $24^{o}.54'$ N.

DISCUSSION

The results obtained for various tilted surfaces are quite promising for the application of solar collectors and equipment at Karachi. The tilt angle calculations have also been done by different authors (7, 8, 9) for other places, suggesting an optimum angle of $\phi + 10^{\circ}$ for the heating season and $\phi - 10^{\circ}$ for the cooling season. In our case a tilt angle of $\phi \pm 10^{\circ}$ shows a variation of only 2% from our suggested value, which is not a very significant change. The selection of an angle $\phi \pm 10^{\circ}$ or $\phi \pm 20^{\circ}$ is not very critical and depends upon many factors (like economy, material cost and other construction parameters).

The results obtained here have been calculated for Karachi for the first time, and give a guideline for designing and orientating solar collectors for optimum utilization of solar energy.

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