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Load Distribution of Heat Source in Production of Heat and Electricity

Pavel Navratil*[1], Libor Pekar*, and Jindrich Klapka [+]

Abstract – *The economic control of the subsystem of heat energy production consists both in economical distribution of load between individual cooperating sources inclusive of economical distribution of load between individual production units inside the sources, for so called operational variant in the source, and further also in determining suitable operational structure of cooperating sources and in determining suitable economically based starting and stopping production units or possibly also the complete sources. The aim of the paper is description one of solution of optimal distribution of load between cooperating production units in the source of heat to optimization of production of heat and electricity. It is created of non-linear mathematical model of heat source for production of heat and electricity. It was used historical and modified data, from the heat and power plant Brno, to creation the mathematical model. The mathematical model is used to determination of load distribution of individual production units and further to determination of the so called dependent and independent electricity that is produced by individual turbo-generator. Described problem of optimization is solved via chosen method of optimization.*

Keywords – heat and electricity, mathematical model, optimization, production unit, turbine.

1. INTRODUCTION

District heating systems (DHS) are created in cities, depending on their size. District heating system should ensure supply of energy to all consumers at minimum costs and with respect to other important aspects, *e.g.* safety, reduction of greenhouse effect gases. Features of DHS are given by its locality and therefore, it is necessary to design a control strategy for each locality [1]. DHS is used in cities of some countries of Europe, North America and Asia, *e.g.* in Czech Republic, Poland, Germany, Austria, France, Denmark, Norway, Sweden, Iceland (there is used mainly geothermal energy), Russia, United State, Canada, Japan and others.

Control of DHS can be considered as control of technological chain which contain three basic sections (see Figure 1), *i.e.* heat production, transport + heat distribution and heat consumption.

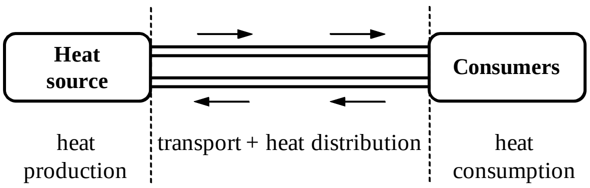

Fig. 1. Technological chain of the DHS.

Technological chain includes heat sources with combined heat and power production (CHP), control system of heat consumption, analysis of static and dynamic behaviour of heat networks to operation control of these networks [2], [3], elimination of influence of transport delay at transport of heat in heat networks [4], [5]. The following part of the paper is focuses on the section "heat production", where is solved an optimization of load distribution of heat source.

Heating plants usually contain several cooperating production units and it is problem of economical load distribution between these ones. The knowledge of the so called consumption characteristics of individual production facilities, *i.e.* boilers and turbines, is the basic requirement of economical production. Therefore, basic task is determination of optimal load distribution between production units. One of possible cases can be minimization the consumption of fuel for required heat output, *i.e.* for heat output delivered to heat network and/or maximization of production of electric energy from individual turbines. The aim of solution can thus be minimization of objective function, *i.e.* minimization of costs of production and/or maximization of objective function, *i.e.* maximization of produced electric energy.

Creation and optimization of non-linear mathematical model of the heat source with combined production of heat and electricity is described in the following text. Non-linearity of mathematical model is given by the non-linear courses of consumption characteristics boilers. The non-linear mathematical model is used for determining of load distribution of individual production units (boilers) and for determining of dependent and independent electric output produced by individual turbines. Optimal parameters of the non-linear mathematical model can be generally determined, for given input conditions, by using chosen optimization algorithm, *e.g.* genetic algorithm [6], [7], pattern search algorithm [8], particle swarm optimization algorithm (PSO algorithm) [9], ant colony algorithm [10], simulated annealing algorithm [11], minimax algorithm [12], goal attainment algorithm [13], etc.

All simulation experiments with created the non-linear mathematical model were carry out via the MATLAB/SIMULINK software [14] via "Optimization

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toolbox" [12], [15]. The MATLAB software is tool for pro-gramming, technical calculation and visualization. The SIMULINK software is a segment of the MATLAB software. It is used to analysing, modelling and simulation of linear and non-linear dynamics systems. The SIMULINK software includes among others also tools for modelling and simulating multidomain physical systems (Physical Modelling tools), such as mechanical, hydraulic, electrical systems, etc. The MATLAB/SIMULINK software is tool which is very used in education but also in research [16], [17].

2. DESCRIPTION OF THE HEAT SOURCE

In this paper, we consider an example of combined production of heat and electricity. The technological scheme of the heat source is shown in Figure 2. Basic information of the scheme and their parameters were obtained from heat and power plant Brno [18], [19] and modified from accessible information. Obtained and modified parameters were used to build the non-linear mathematical model.

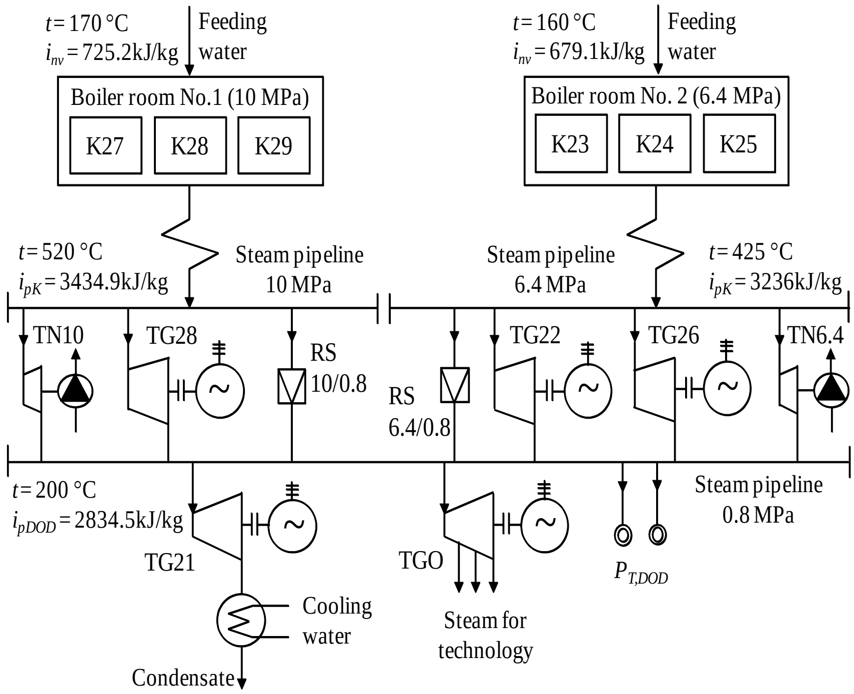

Fig. 2. Technological scheme of combined production of heat and electricity.

The considered heat source (see Figure 2) contains two individual boiler-rooms which have different operating pressure of admission steam, *i.e.* Boiler room No.1 (10 MPa) and Boiler room No. 2 (6.4 MPa). Each of boiler-rooms has three boilers which cooperate into a common steam pipeline. The steam is withdrawn from both high-pressure steam pipelines, marked via operating pressure 10 MPa and 6.4 MPa, by given number of back-pressure steam turbines (TG), turbo-feeders (TN) and reduction stations (RS). From the low-pressure steam pipeline 0.8 MPa steam is delivered through the pipeline network up to consumers. In the steam pipeline network the supplied heat output $P_{T,DOD}$ and the total electric output P are determined. Supplied heat output $P_{T,DOD}$ represents independent variable.

The described mathematical task can be formulated using a linear mathematical model [19] or a non-linear mathematical model. One of the possible solutions described by the mathematical task formulated by the nonlinear mathematical model was described in [18] where the solution of the nonlinear optimization problem was solved by solving a linear optimization problem. This paper uses a nonlinear mathematical model of heat source, generalized here in two-criteria optimization form. In this case, the nonlinear optimization problem solution is performed directly using a nonlinear mathematical model. The nonlinear mathematical model derived in this article is based on the nonlinear mathematical model described in [18]. But in this article, some equations and parameters are slightly modified with respect to a somewhat different view of the problem solved. The basic parameters and characteristics of the separate parts of the heat source mathematical model, *i.e.* the boiler consumption characteristics and the turbine consumption characteristics, are given in Subsection 3.1.

In the next part of text it is described creating of the non-linear mathematical model. The mathematical model is created by using many variables. Generally, symbol labeled $\underline{X}$ represents minimal value (bottom limit) and symbol labeled $\overline{X}$ represents maximal value (upper limit).

3. NON-LINEAR MATHEMATICAL MODEL OF THE HEAT SOURCE

Non-linear mathematical model of combined production of heat and electricity (Figure 2) is created from the knowledge of balance equations of steam piping 10 MPa, 6.4 MPa and 0.8 MPa, and further the objective function (or objective functions) and non-negative limiting conditions of dependent variables. Further it is considered that consumption characteristics of boilers have non-linear (convex) courses (see Figure 3a) and can be expressed using exponential approximation according to (1).

$$P_{T,PAL} = a \cdot e^{\beta P_{T,K}} \quad (1)$$

where $P_{T,PAL}$ is heat output in fuel and $P_{T,K}$ is heat output of boiler.

Further, it is also possible to consider that the turbine consumption characteristics have linear courses (see Figure 3b). The heat output at the turbine inlet $P_{T,T,V}$ is determined by (2) and the heat output at the turbine outlet $P_{T,T,VY}$ is given by the relation (3).

$$P_{T,T,V} = \underline{P}_{T,T,V} + b_{T,V}\Delta P = b_{T,V}P + P_{T,T,V}(0) \quad (2)$$

$$P_{T,T,VY} = \underline{P}_{T,T,VY} + b_{T,VY}\Delta P = b_{T,VY}P + P_{T,T,VY}(0) \quad (3)$$

where $b_{T,V}$ and $b_{T,VY}$ are given positive. P is produced electric output of individual turbines that is in the range $\underline{P}$ (minimal value) up to $\overline{P}$ (maximal value), *i.e.* $\Delta P = \overline{P} - \underline{P}$.

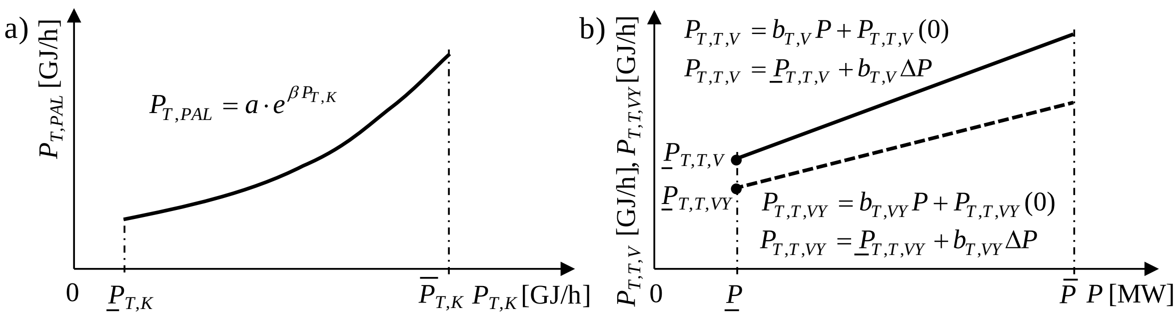

Fig. 3. Basic consumption characteristics: a) boiler, b) turbine.

Table 1. Parameters of boilers K23 and K24.

M_P [t/h]	$P_{T,K}$ [GJ/h]	η [%]	$P_{T,PAL}$ [GJ/h]
50.0	161.73	78.40	163.03
46.0	148.75	78.70	149.58
40.0	129.47	79.00	128.47
34.0	109.78	78.70	109.95
23.8	77.10	77.90	78.14
20.0	64.53	77.40	66.29

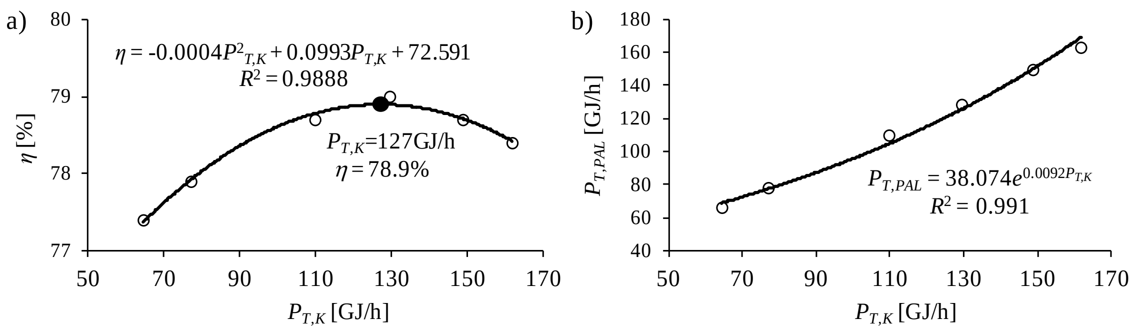

Fig. 4. Characteristics of boilers K23 and K24: a) efficiency curve, b) consumption characteristic.

Table 2. Parameters of boiler K25.

M_P [t/h]	$P_{T,K}$ [GJ/h]	η [%]	$P_{T,PAL}$ [GJ/h]
75.0	243.02	78.30	245.07
71.3	230.87	78.90	230.95
64.2	207.82	79.40	206.78
57.0	184.78	79.20	184.19
50.0	161.73	78.10	162.91

3.1 Basic Characteristics and Parameters of the Heat Source

The non-linear mathematical model of the heat source with combined production of heat and electricity (see Figure 2) was created by using determined consumption characteristics of boilers and turbines (see Figures 4 to 8) and further by using the following tables (see Table 1 to 7). Basic information of parameters of the heat source and corresponding consumption characteristics were obtained from heat and power plant Brno [18], [19] and modified from accessible information.

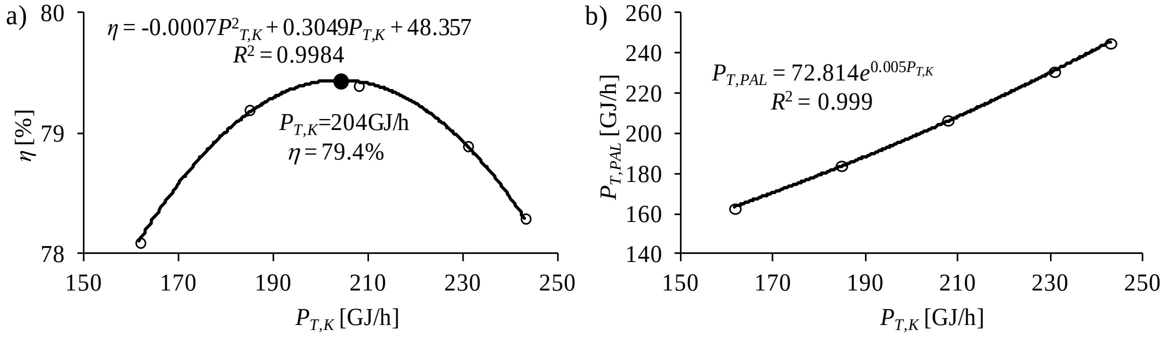

Fig. 5. Characteristics of boiler K25: a) efficiency curve, b) consumption characteristic.

Table 3. Parameters of boilers K27, K28 and K29.

M_P [t/h]	$P_{T,K}$ [GJ/h]	η [%]	$P_{T,PAL}$ [GJ/h]
125.0	429.48	80.30	416.07
100.0	343.58	81.90	331.01
90.0	310.06	82.00	297.49
75.0	257.69	81.40	249.72
60.0	206.57	80.40	201.96
50.0	171.79	79.50	170.74

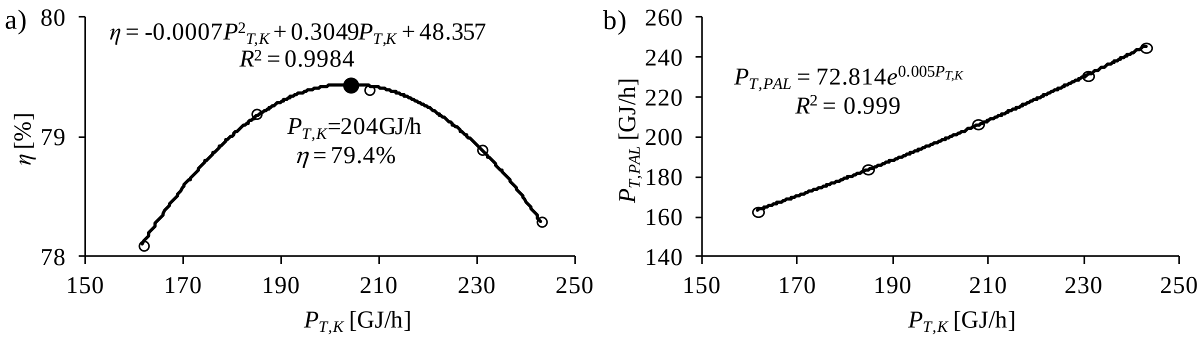

Fig. 6. Characteristics of boilers K27, K28 and K29: a) efficiency curve, b) consumption characteristic.

Table 4. Summary of parameters of individual steam boilers.

Boiler No.	$\underline{M}_P$ [t/h]	$\overline{M}_P$ [t/h]	$\underline{P}_{T,k}$ [GJ/h]	$\overline{P}_{T,k}$ [GJ/h]
K23 (6.4 MPa)	20.00	45.00	64.95	145.81
K24 (6.4 MPa)	25.00	50.00	80.87	161.73
K25 (6.4 MPa)	50.00	75.00	161.73	243.02
K27 (10 MPa)	60.00	115.00	206.57	398.05
K28 (10 MPa)	50.00	100.00	171.79	343.58
K29 (10 MPa)	50.00	100.00	171.79	343.58

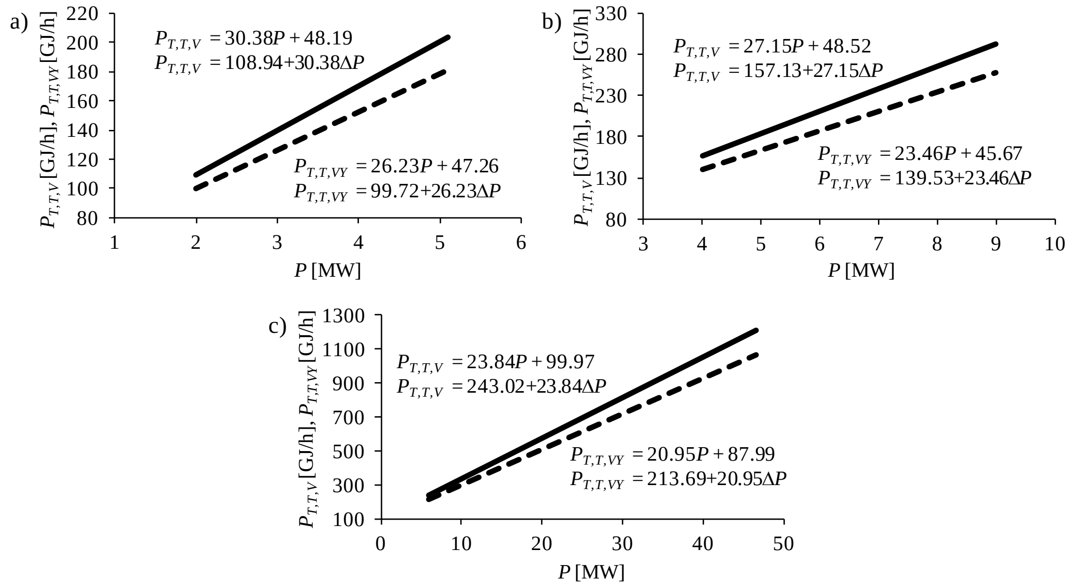

Fig. 7. Characteristics of back-pressure turbine: a) TG22, b) TG26 and c) TG28.

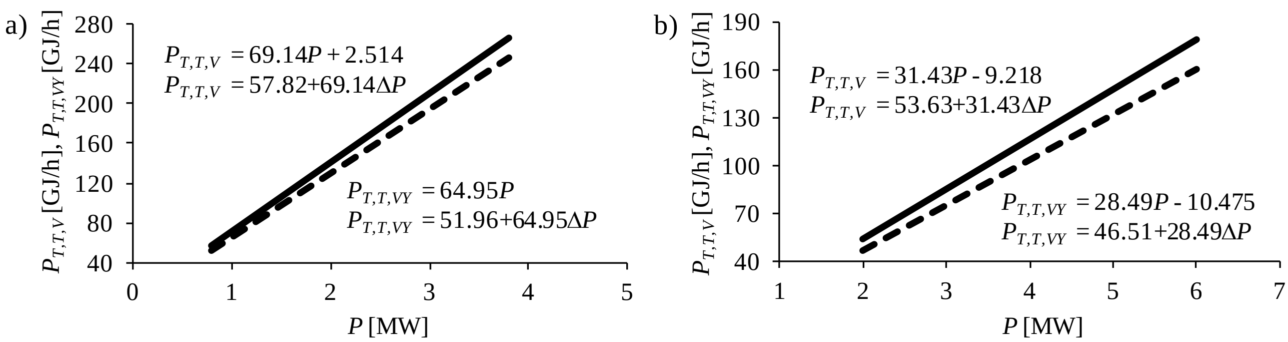

Fig. 8. Turbine characteristics: a) extraction steam turbine TGO, b) condensing turbine TG21.

Table 5. Summary of parameters of individual turbines.

Turbine No.	$\overline{P}$ [MW]	$\underline{P}$ [MW]	$\underline{P}_{T,T,V}$ [GJ/h]	$\underline{P}_{T,T,VY}$ [GJ/h]	$b_{T,V}$ [GJ/MWh]	$b_{T,VY}$ [GJ/MWh]
TG22 (6.4 MPa)	2.0	5.1	108.94	99.72	30.38	26.23
TG26 (10 MPa)	4.0	9.0	157.13	139.53	27.15	23.46
TG28 (10 MPa)	6.0	46.5	243.02	213.69	23.84	20.95
TGO (0.8 MPa)	0.8	3.8	57.82	51.96	69.14	64.95
TG21 (0.8 MPa)	2.0	6.0	55.63	46.51	31.43	28.49

Table 6. Summary of parameters of individual turbo-feeders.

Turbo-feeder No.	$M_{P,N,V}$ [t/h]	$P_{T,N,V}$ [GJ/h]	$P_{T,N,VY}$ [GJ/h]
TN6.4 (6.4 MPa)	15.00	48.60	39.72
TN10 (10 MPa)	15.00	50.91	43.91

Table 7. Summary of parameters of reduction stations.

Reduction station No.	$\overline{M}_{P,R,V}$ [t/h]	$\overline{P}_{T,R}$ [GJ/h]
RS6.4/0.8	120.00	387.99
RS10/0.8	120.00	412.72

3.2 Build the Non-Linear Mathematical Model of the Heat Source

There were considered several the so called operational variants. These operational variants characterize structure of cooperating production units, *i.e.* boilers, turbines, turbo-feeders, reduction stations (see Figure 2). From the possible operation variants, one operational variant was chosen. Non-linear mathematical model is built for the chosen operational variant. There are considered the following conditions:

- Reduction stations (RS), *i.e.* RS6.4/0.8 and RS10/0.8 will not be used.
- Boiler No. K23 and boiler No. K24 have the same efficiency curves and consumption characteristics. Consumption characteristic and efficiency curve of the boiler No. K25 is known.
- The boilers No. K27 - K29 have same efficiency curves and consumption characteristics.
- The electric output of extraction steam turbine (TGO) have two constant values, in the summer time $P = 2$ MW and in the winter time $P = 3.5$ MW. In the non-linear model will be used their arithmetical average, *i.e.* $P = P^{0.8,1} = P_{TGO} = 2.75$ MW.
- If the consumed output condensing turbine (TG21), *i.e.* $P^{0.8,2}$ or P_{TG21}, will be to 2 MW then this consumed output is called dependent electric output because the produced condensate had to be used for cooling the medium at steam production. In case that the consumed output of the TG21 will be in the range from 2 MW to 6 MW then this consumed output is called independent electric output. Increasing of independent electric output of the TG21 from 2 MW up to 6 MW is not only the change of independent electric output of the TG21, but this change causes also production of independent electric output on turbines TG22, TG26 and TG28.
- Non-linear mathematical model of the heat source (see Figure 2) for chosen operational variant is created via heat balance equations of steam piping (4) - (6), which are labeled 10 MPa, 6.4 MPa, 0.8 MPa, objective functions E_1 (7) and E_2 (8), and limiting non-negative conditions (9).

Balance equation of steam piping

- for 10 MPa ($m = 1$, $n = 3$)

$$\sum_{i=1}^{n,10} P_{T,K}^{10,i} = \left[\left(\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{10,j} + b_{T,V}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right)\right) + P_{T,N,V}^{10} + P_{T,R}^{10/0.8}\right] k_q^{10}$$
$$P_{T,K}^{10,1} + P_{T,K}^{10,2} + P_{T,K}^{10,3} = \left[\left(243.02 + 23.84\left(P^{10,1} - 6\right)\right) + 50.91 + 0\right] 1.015 \quad (4)$$

- for 6.4 MPa ($m = 2$, $n = 3$)

$$\sum_{i=1}^{n,6.4} P_{T,K}^{6.4,i} = \left[\left(\sum_{j=1}^{m}\left(\underline{P}_{T,T,V}^{6.4,j} + b_{T,V}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right)\right) + P_{T,N,V}^{6.4} + P_{T,R}^{6.4/0.8}\right] k_q^{6.4}$$
$$P_{T,K}^{6.4,1} + P_{T,K}^{6.4,2} + P_{T,K}^{6.4,3} = \left[\left(108.94 + 30.38\left(P^{6.4,1} - 2\right)\right) + \right.$$
$$\left. + \left(157.13 + 27.15\left(P^{6.4,2} - 4\right)\right) + 48.6 + 0\right] 1.015 \quad (5)$$

- for 0.8 MPa ($m_1 = 1$, $m_2 = 2$, $m_3 = 2$, $n = 3$)

$$\sum_{j=1}^{m_1,10}\left(\underline{P}_{T,T,VY}^{10,j} + b_{T,VY}^{10,j}\left(P^{10,j} - \underline{P}^{10,j}\right)\right) + \sum_{j=1}^{m_2,6.4}\left(\underline{P}_{T,T,VY}^{6.4,j} + b_{T,VY}^{6.4,j}\left(P^{6.4,j} - \underline{P}^{6.4,j}\right)\right) +$$
$$+ P_{T,N,VY}^{10} + P_{T,N,VY}^{6.4} + P_{T,R}^{10/0.8} + P_{T,R}^{6.4/0.8} =$$
$$= \left[\sum_{j=1}^{m_3,0.8}\left(\underline{P}_{T,T,V}^{0.8,j} + b_{T,V}^{0.8,j}\left(P^{0.8,j} - \underline{P}^{0.8,j}\right)\right) + P_{T,DOD}\right] k_q^{0.8}$$
$$213.69 + 20.95\left(P^{10,1} - 6\right) + \left(99.72 + 26.23\left(P^{6.4,1} - 2\right) + 139.53 + 23.46\left(P^{6.4,2} - 4\right)\right) +$$
$$+ 43.91 + 39.72 + 0 + 0 =$$
$$= \left[\left(57.82 + 69.14(2.75 - 0.8) + 53.63 + 31.43\left(P^{0.8,2} - 2\right)\right) + P_{T,DOD}\right] 1.066$$

(6)

- E_1, *i.e.* min E_1, ($n_1 = n_2 = 3$)

$$E_1 = \sum_{i=1}^{n_1,10} a_{10,i}\left[e^{\beta_{10,i} P_{T,K}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}\right] + \sum_{i=1}^{n_2,6.4} a_{6.4,i}\left[e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}\right]$$
$$E_1 = 99.057\left[e^{0.0034 P_{T,K}^{10,1}} - e^{0.0034 \cdot 206,57}\right] + 99.057\left[e^{0.0034 P_{T,K}^{10,2}} - e^{0.0034 \cdot 171,79}\right] +$$
$$+ 99.057\left[e^{0.0034 P_{T,K}^{10,3}} - e^{0.0034 \cdot 171,79}\right] + 38.074\left[e^{0.0092 P_{T,K}^{6.4,1}} - e^{0.0092 \cdot 64,95}\right] +$$
$$+ 38.074\left[e^{0.0092 P_{T,K}^{6.4,2}} - e^{0.0092 \cdot 80,87}\right] + 72.814\left[e^{0.005 P_{T,K}^{6.4,3}} - e^{0.005 \cdot 161,73}\right]$$

(7)

- E_2, *i.e.* max $E_2 \rightarrow \min(-E_2)$, ($m_1 = 1$, $m_2 = 2$)

$$E_2 = \sum_{j=1}^{m_1,10}\left[P^{10,j} - \underline{P}^{10,j}\right] + \sum_{j=1}^{m_2,6.4}\left[P^{6.4,j} - \underline{P}^{6.4,j}\right]$$
$$E_2 = (P^{10,1} - 6) + \left((P^{6.4,1} - 2) + (P^{6.4,2} - 4)\right) = P^{10,1} + P^{6.4,1} + P^{6.4,2} - 12$$

(8)

Non-linearity of the described problem is in a difference of variables in (7), *i.e.* in a difference of expressions $e^{\beta_{10,i} P_{T,K}^{10,i}} - e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}$ and $e^{\beta_{6.4,i} P_{T,K}^{6.4,i}} - e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}$. Values of $e^{\beta_{10,i} \underline{P}_{T,K}^{10,i}}$ and $e^{\beta_{6.4,i} \underline{P}_{T,K}^{6.4,i}}$ are considered as constants, because parameters $\beta_{10,i}, \beta_{6.4,i}$ and $\underline{P}_{T,K}^{10,i}, \underline{P}_{T,K}^{6.4,i}$ are constants.

Limiting non-negative conditions:

$$\underline{P}_{T,K}^{k,i} \le P_{T,K}^{k,i} \le \overline{P}_{T,K}^{k,i} \quad (k = 10, 6.4;\ i = 1, 2, ..., m_k)$$
$$\underline{P}^{t,j} \le P^{t,j} \le \overline{P}^{t,j} \quad (t = 10, 6.4, 0.8;\ j = 1, 2, ..., n_t)$$
$$64.95 \le P_{T,K}^{6.4,1} \le 145.81 \ [\text{GJ/h}], \quad 206.57 \le P_{T,K}^{10,1} \le 398.05 \ [\text{GJ/h}]$$
$$80.87 \le P_{T,K}^{6.4,2} \le 161.73 \ [\text{GJ/h}], \quad 171.79 \le P_{T,K}^{10,2} \le 343.58 \ [\text{GJ/h}]$$
$$161.73 \le P_{T,K}^{6.4,3} \le 243.02 \ [\text{GJ/h}], \quad 171.79 \le P_{T,K}^{10,3} \le 343.58 \ [\text{GJ/h}]$$
$$2 \le P^{6.4,1} \le 5.1 \ [\text{MW}], \quad 6 \le P^{10,1} \le 46.5 \ [\text{MW}]$$
$$4 \le P^{6.4,2} \le 9 \ [\text{MW}], \quad 2 \le P_{TG21}^{0.8,2} = P^{0.8,2} \le 6 \ [\text{MW}] \quad (9)$$

Required value of heat supply $P_{T,DOD}$ [GJ/h] is in the range 300 - 1000 GJ/h in step of 50 GJ/h. Further, it is possible to change independent electric output of condensing turbine (TG21) in the range 2 - 6 MW in step of 1 MW.

Total produced electric output P of all turbines ($m_1 = 1$, $m_2 = 2$, $m_3 = 2$):

$$P = \sum_{j=1}^{m_1,10} P^{10,j} + \sum_{j=1}^{m_2,6.4} P^{6.4,j} + \sum_{j=1}^{m_3,0.8} P^{0.8,j} = \\ P = P^{10,1} + P^{6.4,1} + P^{6.4,2} + 2.75 + P^{0.8,2} \tag{10}$$

The heat output reserve of individual boilers is the amount of heat output that can be used to increase the delivered $P_{T,DOD}$ heat output and also to increase the total electrical power output P. For these purposes, it is then possible to modify relations (4) - (8) as follows, that is, it will be considered $P_{T,K}^{10,1} = \overline{P}_{T,K}^{10,1}$, $P_{T,K}^{10,2} = \overline{P}_{T,K}^{10,2}$, $P_{T,K}^{10,3} = \overline{P}_{T,K}^{10,3}$, $P_{T,K}^{6.4,1} = \overline{P}_{T,K}^{6.4,1}$, $P_{T,K}^{6.4,2} = \overline{P}_{T,K}^{6.4,2}$, $P_{T,K}^{6.4,3} = \overline{P}_{T,K}^{6.4,3}$, which represents maximal heat output of individual boilers, further then $P^{10,1} = P^{\sim,10,1}, P^{6.4,1} = P^{\sim,6.4,1}, P^{6.4,2} = P^{\sim,6.4,2}$ and $P_{T,DOD} = P_{T,DOD}^{\sim}$, whereas $P_x^{\sim}$ are values gained at maximal heat output of individual boilers. The reserve of heat output $P_{T,DOD}^{*}$ is determined from (11) and corresponding reserve of electric output P^* is given by (12):

$$P_{T,DOD}^{*} = P_{T,DOD}^{\sim} - P_{T,DOD} \tag{11}$$

$$\begin{aligned} P^* &= P^{*,10,1} + P^{*,6.4,1} + P^{*,6.4,2} = \left(P^{\sim,10,1} - P^{10,1}\right) + \left(P^{\sim,6.4,1} - P^{6.4,1}\right) + \\ &+ \left(P^{\sim,6.4,2} - P^{6.4,2}\right) = P^{\sim} - \left(P - 2.75 - P^{0.8,2}\right) = \\ &= \left(P^{\sim,10,1} + P^{\sim,6.4,1} + P^{\sim,6.4,2}\right) - \left(P - 2.75 - P^{0.8,2}\right) \end{aligned} \tag{12}$$

where $P_{T,DOD}^{*}$ is determined reserve of heat output at maximal heat output of boilers for given values of $P^{0.8,2}$ and $P_{T,DOD}$, *i.e.* reserve at maximal heat output of boilers is used for increasing supplied heat output; $P_{T,DOD}^{\sim}$ is determined supplied heat output at maximal heat output of boilers for given value of $P^{0.8,2}$; $P_{T,DOD}$ is required value of supplied heat output at given value of $P^{0.8,2}$; P^* is determined reserve of electric output on individual back-pressure turbines at maximal heat output of boilers for given values of $P^{0.8,2}$ and $P_{T,DOD}$, *i.e.* reserve at maximal heat output of boilers is used for increasing production of electric output; $P^{\sim}$ is determined electric output on back-pressure turbines at maximal heat output of boilers for given values of $P^{0.8,2}$ and $P_{T,DOD}$; P (10) is determined produced electric output for given values of $P^{0.8,2}$ and $P_{T,DOD}$.

Thus, a non-linear mathematical model of the heat source is described by means of two objective functions (7), (8), three balance equations (4), (5), (6), ten limiting conditions (9) and twelve variables.

3.3 Determination of Output Parameters of Heat Source

Output parameters of the heat source and their functional dependences are gained via the following procedure, therefore:

1. Determining the supplied heat output $P_{T,DOD}$ from predicted course of the so called Daily Diagram of Heat Supply (DDHS) in the concrete time [20], [21]. In this case it is possible to consider value $P_{T,DOD}$ as a desired value.
2. Determining of the so called "zero point", *i.e.* heat output that is on steam piping 0.8 MPa at TG21 = 2MW (dependent electric output), at TGO = 2.75MW and given $P_{T,DOD}$. The "zero point" represents a boundary between dependent (TG21 = 2MW) and independent electric output (TG21>2MW) at constant value of heat supply $P_{T,DOD}$.
3. Determined results of point No. 2 are assigned on outputs of back-pressure turbines at constant consumptions of heat output by turbo-feeders TN6.4 and TN10.
4. Calculation of heat output of individual boilers $P_{T,K}$ from outputs of point No. 3. Next, it can be possible to calculate the reserve of heat output of individual boilers which is the quantity of heat output that can be used for increasing the supply of $P_{T,DOD}$ and also for increasing production of total electric output P. It means that it is possible to determine reserve of heat output $P_{T,DOD}^{*}$ (11) and corresponding reserve of electric output P^* (12).
5. Gradually increase the consumption TG21 = 2-6MW at step 1MW and given constant value $P_{T,DOD}$ in given time interval. Repeats points No. 3 and No. 4 always at a change of output of TG21. Increasing of independent output of TG21 from 2 MW to 6 MW is not only change of independent electric output to the TG21, but the change causes production also to an independent electric output on the back-pressure turbines TG22, TG26 and TG28. These increases are also included into the independent output P_{NZ} (see the so called "zero point" in the point No.2).

$$\begin{aligned} P_{NZ} &= P - P_{ZV} = \Delta TG21 + \Delta TG28 + \Delta TG22 + \Delta TG26 = \\ &= (TG21 - 2) + (TG28 - TG28_{DEO}) + \\ &+ (TG22 - TG22_{DEO}) + (TG26 - TG26_{DEO}) \end{aligned} \tag{13}$$

where P_{ZV} represents is electric output, *i.e.* electric output produced for the condition TG21 is 2 MW and P is total produced electric output gained via (10). DEO is abbreviation for "dependent electric output", *i.e.* $TG28_{DEO}$, $TG22_{DEO}$, $TG26_{DEO}$ are dependent electric outputs gained at condition TG21 is 2 MW. TG28, TG22, TG26 are electric output determined for condition TG21 is greater than 2 MW.

6. Evaluation of produced independent electric output P_{NZ} (13) and reserve of electric output P^* (12), which can be offered for sale.
7. Determination of output courses, *i.e.*
   - $P_{T,K} = f(P_{T,DOD})$, where electric output TG21 is in the range from 2 MW - 6 MW and the step is 1 MW, whereas $P_{T,K}$ represents heat output of individual boilers K23, K24, K25 and K27, K28, K29 (see Figures 9, 10)
   - $P_{NZ} = f(P_{T,DOD})$ and $P = f(P_{T,DOD})$, where electric output TG21 is in the range from 2 MW - 6 MW and the step is 1 MW, whereas P represents electric output of individual back-pressure turbines TG22, TG26, TG28 and also total produced electric output (see Figures 11, 12, 14)

- $P^{*}_{T,DOD} = f\,(P_{T,DOD})$ and $P^{*} = f\,(P_{T,DOD})$ from reserve of heat output of individual boilers $P_{T,K}$, for individual constant values of electric output of theTG21, where TG21 is 3, 4, 5 and 6 MW (see Figure 13)

3.4 Usage of Created Non-linear Mathematical Model of the Heat Source

Non-linear mathematical model of the heat source described in the Subsection 3.2 is used to optimization requirement basic parameters, *i.e.* heat output $P_{T,K}$ of individual boilers K23-K25 and K27-K29 and electric output P produced on individual back-pressure turbines TG22, TG26 and TG28. The mathematical model is described by balance equations (4) - (6) and objective functions E_1 (7) and E_2 (8). Input parameters of the mathematical model are output parameters of turbines TGO, TG21 and required DDHS $P_{T,DOD}$.

At optimization of determined non-linear mathematic model, there were used the program MATLAB ver. 8.0 (R2012b) [14] specifically of the so called "Optimization toolbox" [12], [15].

The basic version of the MATLAB program code (m-function) is shown below. This program code was used for obtaining optimal parameters of created non-linear mathematical model. The input parameters and corresponding variables of m-function are following:

$$P^{0.8,1} = P_{TGO} = 2.75\,\text{MW} \quad \equiv \quad \text{P081}$$

$$P^{0.8,2} = P_{TG21} = 2\text{ - }6\,\text{MW (step 1 MW)} \quad \equiv \quad \text{P082}$$

$$P_{T,DOD} = 300\text{ - }1000\,\text{GJ/h (step 50 GJ/h)} \quad \equiv \quad \text{Ptdod}$$

Mentioned m-function uses to finding required parameters the so called goal attainment algorithm [13]. This optimization algorithm is implemented in the MATLAB software and uses a method of sequential quadratic programming (SQP) [22] to solution of described problem. The m-function can be used to calculation of optimal parameter of the non-linear model also by using other algorithm of optimization, *e.g.* minimax algorithm [12], genetic algorithm [7], etc. In this case content of code labeled dashed line in the m-file would be replaced by alternative code.

Output courses of function dependences which were determined by using the m-function are shown in the following figures (Figures 9 - 13). Dotted lines in these figures represent minimal and maximal values of heat output $P_{T,K}$ of individual boilers.

```
function [result] = calculation(P081,P082,Ptdod)
%Parameter beta 10,1 up to 6.4,3;
beta=[0.0034;0.0034;0.0034;0.0092;0.0092;0.005];

%Bottom limit of heat output on individual boilers, i.e.
determination Ptk_min
Ptkd101=206.57; Ptkd102=171.79; Ptkd103=171.79;
Ptkd641=64.95; Ptkd642=80.87; Ptkd643=161.73;

%Upper limit of heat output on individual boilers, i.e.
determination Ptk_max
Ptkh101=398.05; Ptkh102=343.58; Ptkh103=343.58;
Ptkh641=145.81; Ptkh642=161.73; Ptkh643=243.02;

%Bottom limit of electric output, i.e.
%determination P_min
Pd101=6.0; Pd641=2.0; Pd642=4.0; Pd081=0.8; Pd082=2.0;

%Upper limit of electric output, i.e. determination P_max
Ph101=46.5; Ph641=5.1; Ph642=9.0;
%Options setting
options = optimset;
options = optimset(options,'Display','off');
options = optimset(options,'FunValCheck','on');
options = optimset(options,'GoalsExactAchieve',2);

%Parameters of separate balance equations, objective
%functions (E=[E1 E2]),determination of upper bounds, bottom
%bounds, start point, goal vector and weighting vector
E=@(p)[(99.057*exp(p(1)*beta(1))+99.057*exp(p(2)*beta(2))
+99.057*exp(p(3)*beta(3))+38.074*exp(p(4)*beta(4))+38.074*ex
p(

p(5)*beta(5))+72.814*exp(p(6)*beta(6))+(99.057*(-
exp(beta(1)*Ptkd101)-exp(beta(2)*Ptkd102)-
exp(beta(3)*Ptkd103))-38.074*(exp(beta(4)*Ptkd641))-
38.074*(exp(beta(5)*Ptkd642))-
72.814*(exp(beta(6)*Ptkd643)))),(-1)*((p(7)+p(8)+p(9))-
(Pd101+Pd641+Pd642))];

Aeq=[1 1 1 0 0 0 -23.84*1.015 0 0; 0 0 0 1 1 1 0 -
30.38*1.015 -27.15*1.015;0 0 0 0 0 0 20.95 26.23 23.46];

beq=[((243.02+(23.84*(-Pd101))+50.91)*1.015);
(((108.94+30.38*(-Pd641))+(157.13+27.15*(-Pd642))+48.6)
*1.015);((((57.82+69.14*(P081-Pd081)+53.63+31.43*(P082-
Pd082))+Ptdod)*1.066)-(213.69+20.95*(-Pd101)+99.72+ 26.23*(-
Pd641)+139.53+23.46*(-Pd642)+43.91 +39.72))];

lb=[Ptkd101;Ptkd102;Ptkd103;Ptkd641;Ptkd642;Ptkd643;
Pd101;Pd641;Pd642];
ub=[Ptkh101;Ptkh102;Ptkh103;Ptkh641;Ptkh642;Ptkh643;
Ph101;Ph641;Ph642];

start_point=lb; goal=[0 0]; weight=[1 1];

%Solution of optimization by chosen algorithm
[y,objective_fun]=fgoalattain(E,start_point,goal,weight,[],[
],Aeq,beq,lb,ub,[],options);


for i=1:1:length(objective_fun)
  if isempty(y) y(1:length(y),i)=-1; end
end

for i=1:1:length(objective_fun)
  if isempty(objective_fun(i)) purpose_fce_sum(i)=-1; end
end

objective_fun(:,2)=(-1)*objective_fun(:,2);

%Heat output of individual boilers K27, K28, K29 and K23,
%K24, K25, i.e. for separate parameters Ptk
Ptk=y(1:6);


%Electric output of individual turbines TG28, TG22, TG26
%from results of solution, i.e. determination of separate
%parameters P
P=y(7:9);

%Output from m-function
result=[P081;P082;Ptdod;objective_fun';Ptk;P]';
```

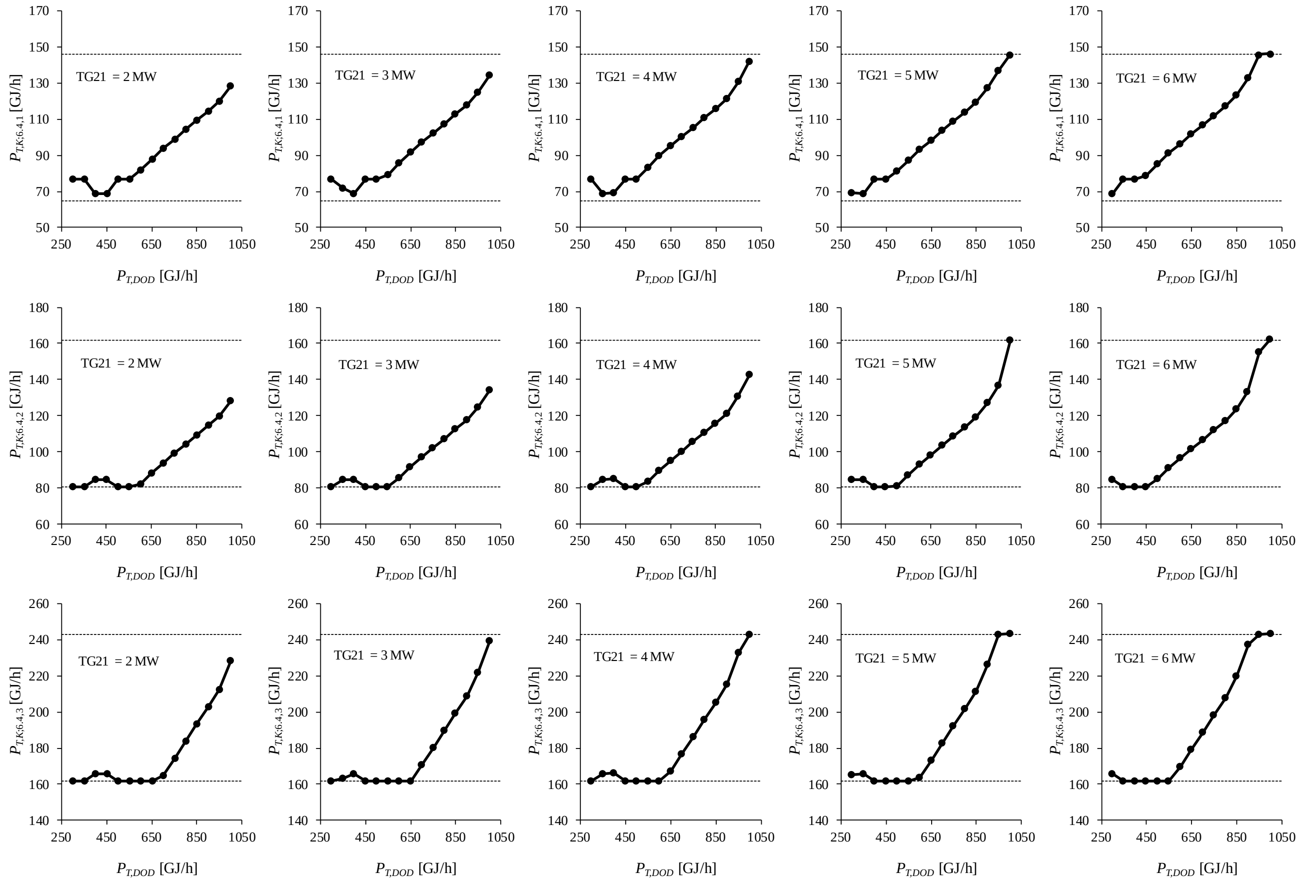

Fig. 9. Optimal load distribution, between cooperating boilers K23, K24, K25, obtained via goal attainment algorithm for required values of electric output on TG21 and supplied heat output $P_{T,DOD}$.

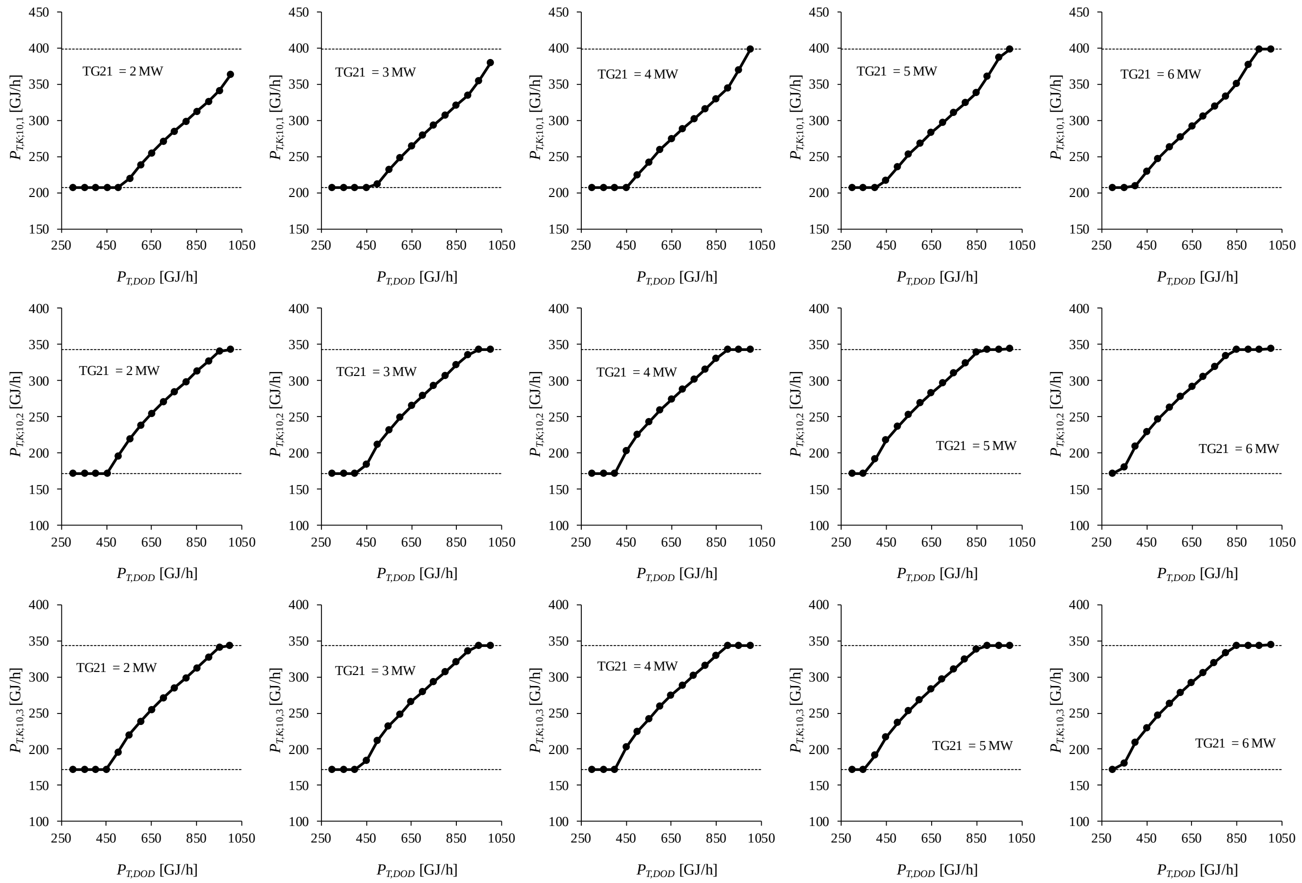

Fig. 10. Optimal load distribution, between cooperating boilers K27, K28, K29, obtained via goal attainment algorithm for required values of electric output on TG21 and supplied heat output $P_{T,DOD}$.

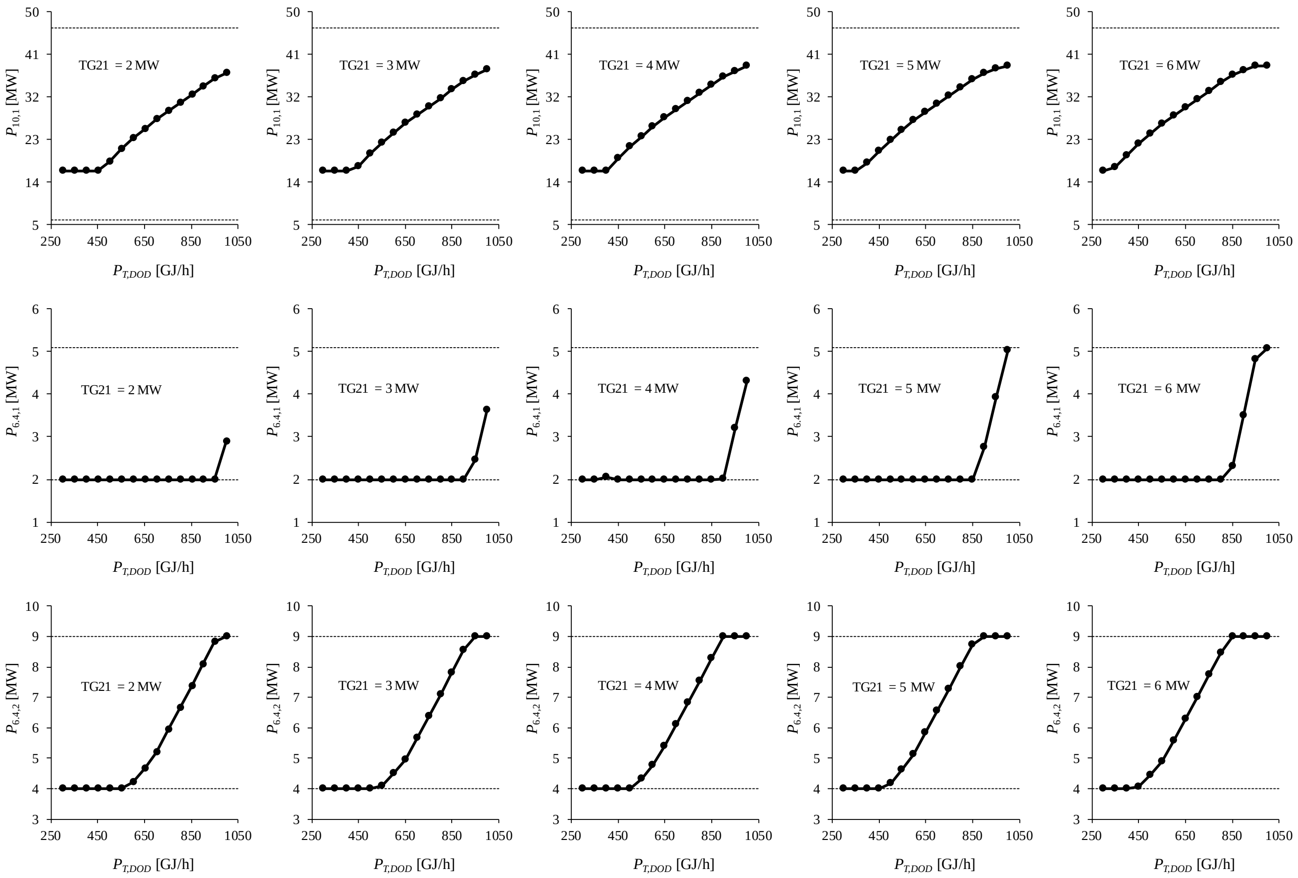

Fig. 11. Output courses electric output of individual back-pressure turbines TG28, TG22, TG26, obtained by optimisation via goal attainment algorithm for required values of electric output on TG21 and supplied heat output $P_{T,DOD}$.

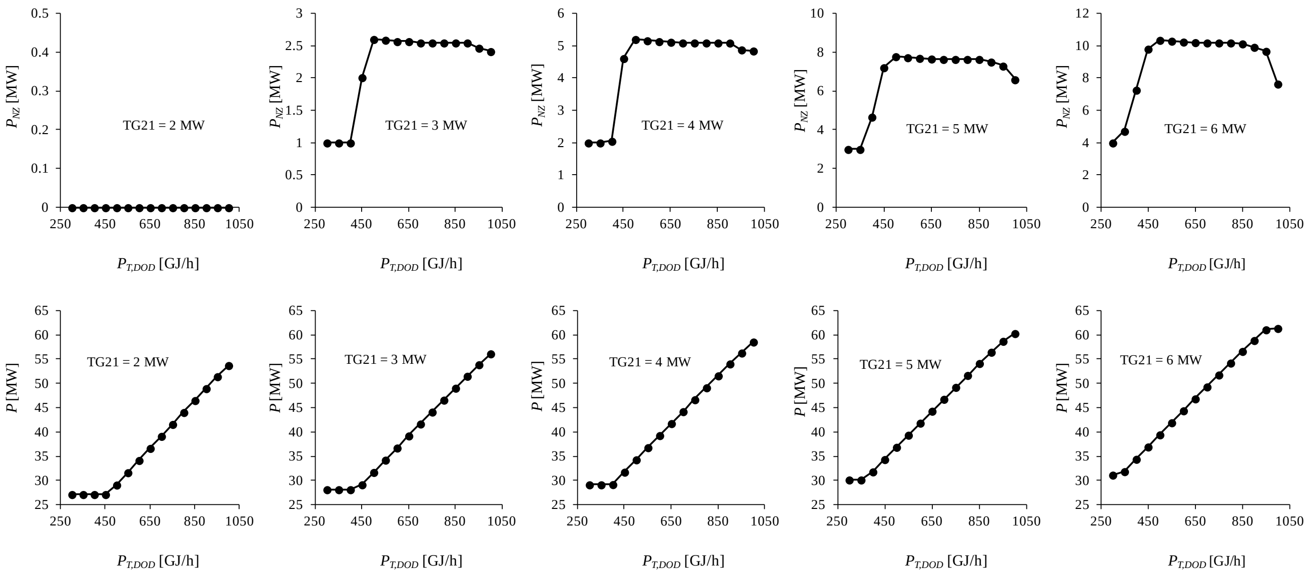

Fig. 12. Output courses independent electric output P_{NZ} and total produced electric output P, obtained by optimisation via goal attainment algorithm for required values of electric output on TG21 and supplied heat output $P_{T,DOD}$.

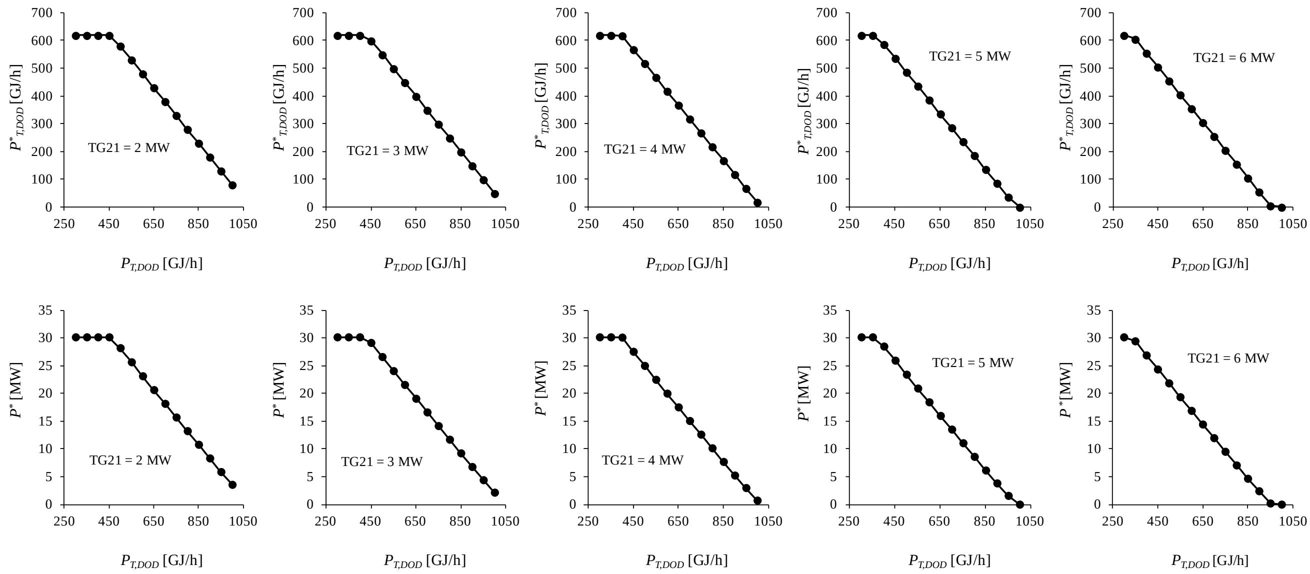

Fig. 13. Output courses of reserve of heat output $P^*_{T,DOD}$ and reserve of electric output P^* on individual back-pressure turbines TG22, TG26, TG28, obtained by optimisation via goal attainment algorithm at required values of electric output on TG21 and supplied heat output $P_{T,DOD}$.

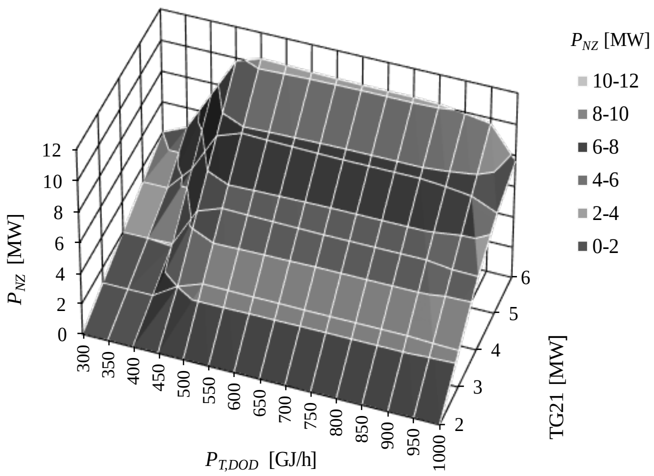

Fig. 14. Output courses of independent electric output P_{NZ}=f($P_{T,DOD}$,TG21); heat supply $P_{T,DOD}$ = 300 - 1000 GJ/h with the step 50 GJ/h, electric output of TG21 = 2 - 6 MW with the step 1 MW.

Table 8. Calculated limited values of heat output $P_{T,DOD}$ at given values of TG21.

TG21 [MW]	$\underline{P}_{T,DOD}$ [GJ/h]	$\overline{P}_{T,DOD}$ [GJ/h]
2	461.59	1080.72
3	430.16	1049.29
4	398.739	1017.86
5	367.30	986.43
6	335.87	955.00

Corrected limited values of heat output $P_{T,DOD}$ and given values of TG21 are presented in the following table (see Table 8). These limited values were calculated at optimization of heat source for considered configuration of cooperating production units (operational variant) by using Equations 4 to 9.

3.5 Evaluation and Possible Way of Utilization of Gained Results

They were shown output courses of consumption characteristics of individual boilers, electric output of individual back-pressure turbines, independent electric output, total produced electric output and also output courses of reserve of heat output and reserve electric output (see Figures 9 to 14). These courses were gained at optimization of heat output described by non-linear mathematical model by using chosen optimization algorithm (see the Subsection 3.4).

The results obtained could be used as a basis for operational planning as well as for real-time operation of heat and electricity production. Therefore, it is possible to use the obtained results as input data for the second part of the district heating system's technological chain (see Figure 1, section "transport + heat distribution"), i.e. for the control of heat output in the district heating system. One of the possible ways of controlling heat output in a district heating system is described in [2], [4], [5], [23]. This method is called the "Qualitative-quantitative control method of hot-water piping heat output with utilization of prediction of the course of heat supply daily diagram in district heating systems". This control method should be able to eliminate the effect of the transport delay between the heat source and the heat consumption of relatively concentrated consumers. The control method of heat distribution consists in the continuous and simultaneous control of two variables (the temperature in the hot water pipeline and the hot water mass flow) affecting the transmitted heat output and the use of the forecast of demand heat output in a specific location [4], [5], [21].

System approach by appropriate (advanced) algorithms to control the whole district heating system's technological chain "heat production, transport + heat distribution and heat consumption" (see Figure 1) should allow the creation of a new higher level of control of DHS. This control of DHS can increase the total efficiency of the district heating system's technological chain, decrease energy prices, increase gains of heat producers and suppliers and increase the level of the environment.

4. CONCLUSION

The aim of this article was to describe one of the possible approaches to optimal load distribution between cooperative production units in a heat source. An example of a heat source with combined heat and electricity was described. The heat source was described by a nonlinear mathematical model. The historical and modified data from the Brno heat and power plant were used to create the mathematical model. This mathematical model was compiled for one selected operating variant. The operational variant is characterized by the composition of cooperating production units, *i.e.* boilers and turbines. The mathematical model created was used to optimize the heat source chosen by the optimization method. The values of the heat outputs of the individual boilers and the reserves of these boilers were determined, *i.e.* the dependent and independent electric output of individual turbines and the reserves of these turbines. The possible use of the obtained results and other possible future work were described in the previous paragraph (see previous Subsection 3.5). Thus, a possible solution of the optimal control of the heat output of the heat source for a specific location with the effect of lowering energy prices and environmental impact was presented and described in the paper.

NOMENCLATURE

$P_{T,PAL}$ heat output in fuel [GJ/h]

P_T heat output [GJ/h]

$P_{T,DOD}$ heat output delivered to heat network, supplied heat output [GJ/h]

$P_{T,K}$ heat output of the boiler [GJ/h], where $P_{T,K} = M_P \cdot i_{pK}$

$P_{T,T}$ heat output of the turbine [GJ/h]

$P_{T,T,V}$, $P_{T,T,VY}$ heat output of the turbine on its inlet/outlet [GJ/h]

P total produced electric output of individual turbines [MW]

M_P mass flow of steam on outlet of boiler [t/h]

$M_{P,N,V}$ mass flow of steam on inlet of feeding turbo-pump [t/h]

$M_{P,R,V}$ mass flow of steam of reduction station [t/h]

$b_{T,V}$, $b_{T,VY}$ specific increment of heat consumption on inlet/outlet to turbine [GJ/MWh]

i_{pK} enthalpy of water steam on outlet of boilers [kJ/kg]

i_{pDOD} enthalpy of water steam on inlet to consumers [kJ/kg]

η efficiency of steam boilers [%]

$P_{T,K}^{r,i}$ heat output of boiler for steam pipeline 6.4 MPa and 10 MPa [GJ/h]; $r = 6.4, 10$; where $P_{T,K} = M_P \cdot i_{pK}$

$P_{T,T,V}^{r}$, $P_{T,T,VY}^{r}$ heat output on inlet/outlet of back-pressure turbine for steam pipeline 6.4 MPa and 10 MPa [GJ/h]; $r = 6.4, 10$

$P_{T,N,V}^{r}$, $P_{T,N,VY}^{r}$ heat output on inlet/outlet of feeding turbo-pump for steam pipeline 6.4 MPa and 10 MPa [GJ/h]; $r = 6.4, 10$

$P_{T,R}^{r/0.8}$ heat output of reduction station for steam pipeline 6.4 MPa and 10 MPa [GJ/h]; $r = 6.4, 10$

k_q^s coefficients of heat losses in separate parts of technological scheme (see Fig. 2) [-]; $s = 0.8, 6.4, 10$

$P^{s,j}$ production of electric energy in terminal condensing turbine or in extraction turbine (steam pipeline 0.8 MPa) and in back-pressure steam turbine (steam pipeline 6.4 MPa and 10 MPa) [MW]; $s = 0.8, 6.4, 10$

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