Optimal Power Dispatch Using Different Fuzzy Constraints in Power Systems

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Abstract – This paper presents comparison studies between different fuzzy models to solve the fuzzy-based optimal power dispatch (OPD) problem. The proposed fuzzy-based OPD model handles fuzzy objectives and constraints and is aimed to obtain the optimal operational settings of system generation outputs. These settings minimize the total generation costs and at the same time guarantees that the power flows in critical lines are less than their maximum limit. The comparison studies are performed considering the changes in fuzzy constraints as membership models. The fuzzy constraints are modeled using two linear fuzzy models, namely triangular and trapezoidal models. Numerical studies are performed based on the fuzzy linear programming (FLP) optimization technique. These studies show that, the changes in membership models have a great effect in generation settings, elimination the overflows in the critical lines, and minimizing the total generation costs.

Keywords – Fuzzy linear programming, linear fuzzy models, optimal power dispatch, transmission bending limits.

1. INTRODUCTION

More than four decades ago, the generalized nonlinear programming formulation of the economic dispatch problem was introduced including voltage and other operating constraints. This formulation was named as the optimal power flow (OPF) problem. The OPF problem plays an important role in power system planning and operation. The OPF problem can be viewed as a process aiming at determining the combination of generation units, which minimizes the total operational costs. Where, identifying the best generation values subject to operational and security constraints is driven by economic techniques. The conventional OPF is formulated as an optimization problem with crisp constraints. The constraints can be classified into a set of equality (power flow equations) and inequality constraints (limits and variables). The inequality constraints are the limits of the control variables and operating limits of power systems. However, in practice, there are two types of inequality constraints: hard constraints and soft constraints. For example, the limits of the generating unit outputs are hard constraints because there are physical limitations on the capacity of the generating units to produce active power. The hard constraints expression means that the physical limitation of the generation units cannot be violated. The fuzzy modeling of power generation outputs is aimed to find the optimal operational settings within their minimum and maximum operational generation limits. On the other hand, the limits for the critical transmission line power flows are soft. Small violations of these limits sometimes are acceptable, especially during stressed situations of the system (e.g. emergency or peak loaded). Identifying any transmission line as critical transmission line is based on the line sensitivity factors to different power system events, line-loading factor, and the line importance in the system operation (Line priority).

Reference [1] solved the OPF problem using the LP technique. Security studies are presented in References [2]–[4]. Lu and Unum in [5] used an interior point algorithm to solve the network constrained security control. A common trend in previous techniques has been towards utilizing fixed values, which may leads to an overestimated solution.

From an operational point of view, minimizing generation cost does not mean that a rigid minimum solution is achieved. It is more appropriate to state the OPD objectives as: to reduce the generation costs as much as possible without moving too many control settings, while satisfying the soft constraints as much as possible and enforcing the hard constraints exactly. Here, the concepts of “as much as possible” and “not too many” are fuzzy in nature. Fuzzy logic has found favour among many engineers for its ability to represent the sorts of qualitative statements employed by human. The conventional logic assumes that a variable has one precise value (it is crisp).

Recently, fuzzy set methods have been applied to obtain realistic models. Fuzzy set methods have already been used in many applications such as control, scheduling, robotics, artificial intelligence, etc. In the field of power system engineering, fuzzy set methods have been applied to some areas including OPF problems. References [6]–[10] presented the solution of the optimal power flow problem using the FLP technique. Reference [11] solved OPD problem considering multi-objective FLP technique considering preventive action constraints. Different emergency control analyses procedure using multi-objective FLP technique are presented in [12].
This paper presents a fuzzy-based OPD procedure taking into account fuzzy modeling for both equality and inequality constraints. Two linear fuzzy models (triangular and trapezoidal models) are used to model the power system variables. The fuzzy constraints improve the OPD solution as:

- Finding the optimal operational settings of these variables within the operational generation range;
- Tuning the power systems variables;
- Ramping the power generation and power transmission lines fuzzy constraints corresponding to the amount of reserve requirements; and
- Considering the uncertainty in power systems.

2. PROBLEM FORMULATION

Conventional Optimal Power Dispatch

The objective of the conventional OPD problem is to minimize the total generation costs under various system and operational constraints. The OPD problem is formulated as:

\[
\text{Min } C = \sum_{i=1}^{NG} F_i \left( PG_i \right)
\]

Subject to:

- Power balance constraint.

\[
\sum_{i=1}^{NG} PG_i = PD
\]

- Power flow constraint

\[
PF_k^{\text{min}} \leq PF_k \leq PF_k^{\text{max}}
\]

The power flow \( PF_k \) can be computed from:

\[
PF_k = \sum_{i=1}^{NG} D_{i,k} PG_i, \text{ for } i = 1, \ldots, NG
\]

Where, \( D_{i,k} \) is the generalized generation distribution factor for line \( k \) due to generator \( i \) [13].

- Power generation limits constraints.

\[
PG_i^{\text{min}} \leq PG_i \leq PG_i^{\text{max}}
\]

Fuzzy Optimal Power Dispatch Problem

The fuzzy-based OPD problem is formulated as:

\[
\text{Min } C = \sum_{i=1}^{NG} F_i \left( \hat{PG}_i \right)
\]

Subject to:

\[
\sum_{i=1}^{NG} \hat{PG}_i = \hat{PD}
\]

\[
PF_k^{\text{min}} \leq \hat{PF}_k \leq PF_k^{\text{max}}
\]

\[
PG_i^{\text{min}} \leq \hat{PG}_i \leq PG_i^{\text{max}}
\]

Linear Fuzzy Models Formulation

Before starting with the fuzzy modeling of constraints, it is important to define the meaning of the considered models, which are the trapezoidal model and the triangular model. Let the symbol \( P \) be used to express one of these constraints. For instance through a linguistic declaration as “power \( P \) may occur between \( P_1 \) and \( P_3 \) MW but likely to be between \( P_2 \) and \( P_3 \). This can be translated into trapezoidal fuzzy model at which the uncertainty through interval. If \( P_2 = P_3 \), the resulted model will be define the triangular model of power constraints. The next two sections deal with the two fuzzy models of the power system constraints.

Triangular fuzzy modeling

The triangular fuzzy modeling for the active power generation at bus \( i \) is shown in Figure 1a. It is seen that, a membership function is equal to 1 assigned to \( PG_i^{\text{med}} \). The triangular fuzzy modeling for the power flow in critical line \( k \) is shown in Figure 1b. It is seen that, a membership function is equal to 1 assigned to \( PF_k^{\text{med}} \). The triangular membership functions, for generation limit at bus \( i \) and for the power flow in critical transmission line \( k \), are presented in Equations 10 and 11, respectively.

\[
\mu[PG_i] = \begin{cases} 1, & \text{for } PG_i = PG_i^{\text{med}} \\ 0, & \text{otherwise} \end{cases}
\]

\[
\mu[PF_k] = \begin{cases} 1, & \text{for } PF_k = PF_k^{\text{med}} \\ 0, & \text{otherwise} \end{cases}
\]

Trapezoidal triangular membership model

The trapezoidal fuzzy modeling of the power generation and the power flow in critical lines constraints are presented in Figure 2. The trapezoidal membership functions of the power generation at bus \( i \) and the power flow in the critical transmission line \( k \), the violated transmission line, considered as critical line, are described and shown in Equations 12 and 13, respectively.

\[
\mu[PG_i] = \begin{cases} 1, & \text{for } PG_i = PG_i^{\text{med}} \\ 0, & \text{otherwise} \end{cases}
\]

\[
\mu[PF_k] = \begin{cases} 1, & \text{for } PF_k = PF_k^{\text{med}} \\ 0, & \text{otherwise} \end{cases}
\]
**Fuzzy Modeling of Load Demand**

Similarly, for the fuzzy modeling of load demand, Figures 3a and 3b show the triangular and the trapezoidal membership function for load demand, respectively. The triangular and trapezoidal membership functions for the load demand are described in Equations 14 and 15, respectively.

**Fuzzy Modeling of Objective Function**

The objective function which is considered in the proposed procedure minimized the generation cost function as much as possible. The fuzzy modeling of the generation cost function is shown in Figure 4. The fuzzy membership function of the cost, which is less than or equals the permissible cost, is described in Equation 16.

\[
\begin{align*}
\mu_{PG_i}(PG_i) &= \begin{cases} 
0 & \text{if } PG_i < PG_i^{med} \\
\left(PG_i - PG_i^{min}\right)\left(PG_i^{med} - PG_i^{min}\right) & \text{if } PG_i^{min} \leq PG_i \leq PG_i^{med} \\
\left(PG_i^{max} - PG_i^{med}\right)\left(PG_i^{max} - PG_i^{med}\right) & \text{if } PG_i^{med} \leq PG_i \leq PG_i^{max} \\
0 & \text{if } PG_i > PG_i^{max}
\end{cases}
\end{align*}
\]

(10)

\[
\begin{align*}
\mu_{PF_k}(PF_k) &= \begin{cases} 
0 & \text{if } PF_k < PF_k^{min} \\
\left(PF_k - PF_k^{min}\right)\left(PF_k^{med} - PF_k^{min}\right) & \text{if } PF_k^{min} \leq PF_k \leq PF_k^{med} \\
\left(PF_k^{max} - PF_k^{med}\right)\left(PF_k^{max} - PF_k^{med}\right) & \text{if } PF_k^{med} \leq PF_k \leq PF_k^{max} \\
0 & \text{if } PF_k > PF_k^{max}
\end{cases}
\end{align*}
\]

(11)

\[
\begin{align*}
\mu_{PG_i}(PG_i) &= \begin{cases} 
0 & \text{if } PG_i < PG_i^{min} \\
\left(PG_i - PG_i^{min}\right)\left(PG_i^{(1)} - PG_i^{min}\right) & \text{if } PG_i^{min} \leq PG_i \leq PG_i^{(1)} \\
\left(PG_i^{max} - PG_i^{max}\right)\left(PG_i^{max} - PG_i^{(2)}\right) & \text{if } PG_i^{(1)} \leq PG_i \leq PG_i^{(2)} \\
0 & \text{if } PG_i > PG_i^{max}
\end{cases}
\end{align*}
\]

(12)
3. PROPOSED PROCEDURE FOR OPTIMAL POWER DISPATCH PROBLEM

Linearization of the Generation Cost Function

The OPD with quadratic form of generation cost functions is formulated as nonlinear optimization problem. The solution of the OPD problem using FLP technique requires linear objective function.

The quadratic generation cost function of the form:
\[ F_i(PG_i) = a_i PG_i^2 + b_i PG_i + c_i \]  

The generation cost function, of unit i, in linear form for small variation in unit i power generation output can be written with the help of as the basics of derivative as:
\[ F_i = \frac{dF_i}{dPG_i} \bigg|_{PG_i=0} PG_i \]  

Then, the approximate form of total generation cost function is written as:
\[ F_T = \sum_{i=1}^{N} (2a_i PG_i^2 + b_i)PG_i \]  

FLP Optimization Model

The FLP optimization technique is used to solve the fuzzy-based OPD problem (5-8). The degree of satisfaction the fuzzy objectives and constraints, Equations 10–16, can be represented by a membership variable λ. The variable λ is defined as the minimum of all membership functions of the fuzzy objectives constraints. The fuzzy-based optimal OPD solution maximizes satisfaction variable λ. Then, the relationship between the satisfaction factor λ and other membership functions can be written as the minimum of all membership functions. In this section the fuzzy symbols appeared as the degree of membership function. The mathematical model is:
\[ \max \lambda \]
The procedure steps, for certain studied condition are:

1. Simulating the operating condition.
2. Computing the initial generation settings and the related power flows in transmission lines.
3. Identifying the violated transmission lines as critical transmission lines.
4. Preparing the fuzzy modeling of different system variables based on the initial state.
5. Solving the OPD problem using the proposed procedure.

6. Ensuring the power flows in all transmission lines within their permissible limits.
7. If there is not a violation, print results else modify the critical transmission lines and (Go to step 3).

4. APPLICATIONS

Test System

The IEEE 30-bus test system (6-generation units, 41-lines) [14] is used to extensively study the OPD problem using the FLP procedure for different fuzzy models. The bus data of the six generation units are presented in Table 1 while, the data for 8-critical transmission line is presented in Table 2. These lines are lines No. 1, 2, 4, 5, 6, 9, 11, and 14. The power flow computations are performed using the MATPOWER package [15].
power generation, flows, and power demand. In Case 2, two linear constraints corresponding to the trapezoidal model of load demand are introduced as Equations 40 and 41 instead of Equations 35 and 36 in Case 1. The same procedure is followed to ramp the system constraints for satisfying the other cases. Case 8 considers Equations 37 to 41 instead of Equations 31 to 36 in the fuzzy-based OPD problem.

Table 3. Fuzzy membership models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Power Flow</td>
<td>*</td>
<td>*</td>
<td>***</td>
<td>*</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td>**</td>
</tr>
<tr>
<td>Load</td>
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<td>***</td>
<td>0</td>
<td>**</td>
<td>0</td>
<td>*</td>
<td>**</td>
</tr>
</tbody>
</table>

Where (*) Refers to use the triangle model and (**) refers to use the trapezoidal model.

Intermediate points of membership functions

The OPD solution is dependent on the choice of \( PG_{i}^{*} \), \( PF_{k} \) and \( PD_{i} \) in the case of triangular fuzzy modeling. In this paper, the med-points of power generation units and the power demand equal to the initial generation and demand values. While, the med-points of the power flows in transmission lines are considered at 90% of the maximum limit of these lines. Also, The OPD solution is very significant to the choice of the intermediate points of the trapezoidal model of power generation at unit \( i \) (\( PG_{i}^{1} \) and \( PG_{i}^{2} \)), power flow in critical transmission line \( k \) (\( PF_{k}^{1} \) and \( PF_{k}^{2} \)), and the total power demand (\( PD_{i}^{1} \) and \( PD_{i}^{2} \)). These intermediate points are adjusted at 10% and 90% of the variable range, while the variable membership degree is varied from (1–0). The optimal values of these intermediate points are computed according to their effects for achieving the problem objectives and constraints.

5. RESULTS AND COMMENTS

The proposed fuzzy models are applied for solving the OPD problem for normal operation and emergency conditions. The objective functions for normal conditions are minimizing the generation costs and maximizing the power reserve in the critical transmission lines. For the emergency conditions, the previous objectives should be satisfied and the overflows in transmission lines must be eliminated.

The studied conditions are summarized as follows:

Normal operation

Table 4 shows the proposed OPD results for different fuzzy modeling cases at \( \sum_{i=1}^{NG} PG_{i} = 268.9 \) MW. The generation costs are minimized for all different fuzzy models cases compared with conventional LP solution. The maximum reduction in the generation costs was obtained with savings of 48.93 \$/hr which occurred in cases 5 and 6. While, the minimum reduction in generation cost (40.95 \$/hr) occurred in cases 3 and 4. The savings in generation costs in cases 1 and 2 was 42.37 \$/hr. In cases 7 and 8 the total generation costs decrease by 48.07 \$/hr. It is clear that, the fuzzy-based OPD results minimized the total generation costs compared with the conventional LP result for all fuzzy modeling cases. Table 5 shows the corresponding power flows in the critical transmission lines. The overflows in the critical transmission lines are fully removed. These tables present different reserve levels obtained for power generation units and from transmission lines. For example, for Line No. 1, the maximum reserve level for this line occurred at the conventional LP solution (30,454 MW). While, the minimum reserve level (2,383 MW) occurred at cases 5 and 6. Cases 1 to 4, 7, and 8 presented different reserve levels for this line as shown in Table 6. It is clear that, the fuzzy-based OPD results increased the total generation costs in proportional manner to the amount of reserves from critical transmission lines.

Effect of load demand variations

Different studied cases are introduced to discuss the load demand as a judgment in the OPD problem. Tables 6 show the total generation costs of the proposed fuzzy-based OPD model at different loading levels with variant maximum transmission limits of critical transmission lines. Table 6 show that, the trapezoidal representation, i.e. case 6, is the best fuzzy model of load demand over the loading interval between 150 to 300 MW. The best generation costs occurred at case 2 for power demand of 150 MW and are at both cases 4 and 8 at loading point of 200 MW. In cases 2, 4, and 8, the trapezoidal representation of power demand was proposed. The power demands at cases 5 and 6 have equal effects. Case 6 is the preferable one for load representation with trapezoidal model. The proposed fuzzy models lead to minimized total generation costs in a manner less than the generation costs that results from the conventional LP. It is seen that, the main benefit of trapezoidal membership model over the triangular fuzzy model is the good distribution of power generation and power demand.

Effect of transmission bending limit variations

The transmission lines power flow ranges in the model were treated as fuzzy constraints. The decrease in transmission lines limits helped us to model the stressed system cases. Transmission limit variations were presented to show these effects in the stressed cases. Table 7 shows the effects of bending limit variation for transmission lines. In this table, the maximum limit of the power flows in critical transmission lines are allowed to increase from 44 MW to 46, 48, 50, and 52 MW. The increase of bending limit did not have an affect on the fuzzy results. The results of fuzzy-based modeling were obtained by fine-tuning of the generation settings. Then, the fuzzy-based solution is still at the best economic level. The LP solution was improved with increasing the bending level as the generation costs were decreased from 861.41 \$/hr to 859.3 \$/hr when increasing the bending limit from 46 to 52 MW.

Tables 4-7 lead to the following comments:

1. The proposed method validated for both normal and emergency conditions as system contingencies and increase in power demand tunes the searching of economic generation settings while the power flows in TLs are away from their bending limits.
2. The generation costs minimized compared with the conventional LP case for all studied fuzzy cases.
3. The use of trapezoidal membership model for the power generations and power demand constraints lead to minimized generation costs. In terms of efficiency, the trapezoidal fuzzy model may be the suitable one for power generation and power demand. The main benefit of trapezoidal membership model over the triangular fuzzy model is the good distribution of power generation and power demand. The trapezoidal membership function achieves the physical operation of generation units. Representation that is more accurate was found based on the trapezoidal model, which suits the modeling of hard constraints of generation units.

4. In terms of efficiency, the trapezoidal fuzzy model may be the suitable one for power generation and power demand. The main benefit of trapezoidal membership model over the triangular fuzzy model is the good distribution of power generation and power demand. The trapezoidal membership function achieves the physical operation of generating units.

5. In terms of efficiency, the use of triangular fuzzy model in the case of transmission power flows leads to more effective use of transmission lines.

6. The use of triangular fuzzy model for the power flows in critical lines reduces the total generation cost compared to the use of trapezoidal model.

7. The proposed fuzzy modeling leads to very variety degree in dealing with different power systems variables.

8. The optimal operational settings of these variables within the operational generation range are obtained.

9. Fine-tuning of power system variables reduce the generation cost than the conventional techniques.

10. Different reserve levels from power generation units and critical transmission lines are satisfied corresponding to the fuzzy models of generation and transmission lines fuzzy constraints.

---

### Table 4. Power generations and generation cost of the OPD for different fuzzy models ($\sum_{i=1}^{NG} PG_i = 268.9$ MW)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Initial Condition</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>LP Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PG_1$</td>
<td>152.98</td>
<td>114.1</td>
<td>114.1</td>
<td>113.52</td>
<td>113.52</td>
<td>117.1</td>
<td>117.1</td>
<td>116.67</td>
<td>116.67</td>
<td>85.877</td>
</tr>
<tr>
<td>$PG_2$</td>
<td>57.56</td>
<td>51.469</td>
<td>51.469</td>
<td>51.32</td>
<td>51.32</td>
<td>51.786</td>
<td>51.786</td>
<td>51.563</td>
<td>51.563</td>
<td>66.595</td>
</tr>
<tr>
<td>$PG_3$</td>
<td>24.56</td>
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<td>33.122</td>
<td>33.642</td>
<td>33.642</td>
<td>30.19</td>
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<td>29.28</td>
</tr>
<tr>
<td>$PG_4$</td>
<td>31.404</td>
<td>32.362</td>
<td>32.362</td>
<td>32.362</td>
<td>32.362</td>
<td>32.358</td>
<td>32.358</td>
<td>32.359</td>
<td>32.359</td>
<td>31.775</td>
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</tbody>
</table>

*Generation Costs $/hr* 818.432 752.93 752.93 754.35 754.35 746.37 746.37 747.23 747.23 795.3

### Table 5. Power flows in critical lines of the OPD for different membership models ($\sum_{i=1}^{N_L} P_{L,i} = 269.8$ MW)

<table>
<thead>
<tr>
<th>Lines</th>
<th>Max. Limits</th>
<th>Initial Condition</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>LP Solution</th>
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<td>1</td>
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<td>69.491</td>
<td>69.491</td>
<td>72.617</td>
<td>72.617</td>
<td>72.26</td>
<td>72.26</td>
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<td>2</td>
<td>50</td>
<td>52.54</td>
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<td>43.791</td>
<td>43.694</td>
<td>43.694</td>
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<td>44.137</td>
<td>44.137</td>
<td>40.502</td>
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<td>40.501</td>
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<td>30.448</td>
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<td>31.223</td>
<td>31.054</td>
<td>31.054</td>
<td>27.432</td>
</tr>
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</table>

### Table 6. Generation costs of the different modeling cases for different loading points

<table>
<thead>
<tr>
<th>Load (MW)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>LP Solutions</th>
</tr>
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<tr>
<td>150</td>
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<td>376.2</td>
<td>389.12</td>
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<td>376.86</td>
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<tr>
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<td>446.65</td>
<td>442.11</td>
<td>448.26</td>
<td>441.41</td>
<td>448.43</td>
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</tr>
<tr>
<td>200</td>
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<td>514.36</td>
<td>518.84</td>
<td>513.41</td>
<td>516.94</td>
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<td>521.38</td>
<td>513.41</td>
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<td>590.76</td>
<td>588.78</td>
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<td>684.19</td>
<td>684.19</td>
<td>675.58</td>
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<td>676.4</td>
<td>676.4</td>
<td>736.34</td>
</tr>
<tr>
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<td>754.35</td>
<td>754.35</td>
<td>746.37</td>
<td>746.37</td>
<td>747.23</td>
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<td>285</td>
<td>817.22</td>
<td>817.22</td>
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<td>812.13</td>
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<td>862.75</td>
<td>862.75</td>
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<td>882.18</td>
</tr>
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</table>
Table 7. Effects of variant bending limit of critical transmission line no. 4

<table>
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<tr>
<th>PF₄ bending limits (MW)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>LP solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>835.18</td>
<td>835.18</td>
<td>835.6</td>
<td>835.6</td>
<td>833.01</td>
<td>833.01</td>
<td>833.29</td>
<td>833.29</td>
<td>861.41</td>
</tr>
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<td>833.29</td>
<td>833.29</td>
<td>859.3</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

This paper presented an efficient and a reliable procedure to solve optimal power dispatch problem in power systems. The proposed procedure minimized the total generation costs and at the same time, eliminated the overflows in critical transmission lines. Comparison studies between the two linear fuzzy models, trapezoidal and triangular membership models, covering many system conditions, helped the programmer in choosing the best model that the operator may use. Trapezoidal membership for modeling both of power generation and power demand constraints lead to minimized generation cost. While, the use of triangular membership model for modeling the power flows in the critical transmission lines leads to minimized generation cost. The proposed procedure can help the operator to ramp the system constraints corresponding to the amount of reserve requirements. The proposed procedure leads to allocate both responsibility and security action payments to system individuals.

NOMENCLATURE

**Control Variables**

- \( PG_i \): generation outputs of unit \( i \) (MW).
- \( PG_i^{\text{min}} \): minimum limit of generation for unit \( i \) (MW).
- \( PG_i^{\text{max}} \): maximum limit of generation for unit \( i \) (MW).
- \( PG_i^{\text{mod}} \): a point within the operational range of generation unit \( i \) (MW).
- \( PG_i^{(0)} \): initial power generator output \( i \) (MW).
- \( PG_i^{(1)} \): a point within the operational range of generation unit \( i \) (MW).
- \( PG_i^{(2)} \): a point within the operational range of generation unit \( i \) (MW).
- \( NG \): number of generation buses.

**Dependent Variables**

- \( PF_k \): power flow in line \( k \) (MW).
- \( PD \): total power demand (MW).
- \( F_i(PG_i) \): generation cost of unit \( i \) ($/hr).
- \( a, b, c \), and \( c \): generation cost coefficient of unit \( i \) ($/hr).
- \( C \): total generation costs of all generation units ($/hr).
- \( PF_k^{\text{min}} \): minimum limit of power flow in critical line \( k \) (MW).
- \( PF_k^{\text{max}} \): maximum limit of power flow in critical line \( k \) (MW).

**Fuzzy Variables**

- \( \tilde{PG}_i \): fuzzy active power generation (MW).
- \( \tilde{PD} \): fuzzy load demand included power losses (MW).
- \( \tilde{PF}_k \): fuzzy active power transmission line flow in line \( k \) (MW).
- \( \mu_{\tilde{PG}_i} \): lower fuzzy membership function for generator \( i \).
- \( \mu_{\tilde{PF}_k} \): lower fuzzy membership function for critical line \( k \).
- \( \mu_{\tilde{PD}} \): lower fuzzy membership function for load demand.
- \( \mu_c(C) \): fuzzy membership function for objective cost function.
- \( NC \): number of fuzzified constraints.

REFERENCES


