Minimize Energy Losses Configuration of Power Distribution System

J. Dhiman* and T. Thakur*

Abstract – The reduction of energy losses in distribution system is an important issue during planning and operation of electrical engineering. In this paper, a new method to minimize energy losses configuration in power distribution system using DLF program is presented. Two matrices that are developed from the topological characteristics of distribution systems are used to solve distribution network problems. These two matrices are combined to form a direct approach for solving reconfiguration problems. This work has been tested on 33-bus system. Merit of used method is that it is very effective and solutions get converged even in seconds.

Keywords – Energy losses, heuristic rule, network reconfiguration.

1. INTRODUCTION

The evaluation and reduction of electric energy losses are important tasks for electrical engineering in power distribution system. The configuration for reduction of the energy losses originates two important saving - one based on the decrease of the energy generation requirements and the other by the decrease of the maximum load equipment requirements [1].

Reducing energy losses configuration in distribution is often a key motivation for the incorporation of the private sector in such systems in many countries around the world [2]-[4]. This is so since it is not uncommon for energy losses ratios in these countries to be very high, oftentimes exceeding 30% or even 40% of purchases. This is compounded by the fact that such utilities often have a hard time collecting what they bill. In some cases, utilities are only able to collect about 50% to 60% of billings, making their effective losses rather high.

Therefore, the challenge is more pronounced in case of distribution systems. Basic reason behind these huge power energy losses is resistive loss, as distribution systems are operated at much lower voltages as compared to transmission systems. So, operating current in distribution system is much more than that in transmission systems, and hence, larger power energy loss (resistive) in distribution system is much more than that in transmission systems [5]. So, in totality, optimal operation of a distribution power networks has become an engineering challenge.

The concept of reconfiguring the topology of the distribution network to minimize energy losses can immediately be recognized as being cost efficient and consequently of interest to efficiency conscious electric utilities [6]. Electric distribution networks are mostly figured as radial for proper protection coordination. Distribution feeders may be frequently reconfigured by opening and closing switches to while meeting all load requirements and maintaining a radial network [7]-[9]. These requirements result in a very complicated non-linear integer optimization problem. The exact optimal configuration solution of such a problem may be obtained only by adumbratively examining all possible switch options requiring prohibitively long computation time because the number of switch options is usually very large in a practical distribution network [10]. Therefore many heuristic approximation methods for reconfiguration have been proposed for efficiently solving the problem.

This paper presents a new method to minimize energy losses configuration in power distribution system. In the process of calculation, two developed matrices Bus Injection to Bus Current (BIBC) and Branch Current to Bus Voltage (BCBV), and a simple matrix multiplication were used to obtain load flow solutions. The solution converged very early on; therefore execution time is very small. The results reveal the speed and the effectiveness of the proposed method for solving the problem.

2. PROBLEM FORMULATION

Volatges at the Buses

In order to obtain energy losses configuration solutions, first objective is to obtain voltages at the buses.

If \( V_k \) is the voltages of the buses after \( k^{th} \) iteration, then voltages at the buses after \( (k+1)^{th} \) iteration is given by:

\[
V^{k+1} = V^k - \Delta V^k
\]

(1)

Here \( \Delta V^k \) is change in bus voltages after two successive iterations.

Real Power Flow

\[
P_{ij} = \text{Real}[V_i (V_i - V_j) y_{ij}^*]
\]

(2)

Here \( P_{ij} \) is the real power flowing through the line connecting \( i^{th} \) and \( j^{th} \) buses, \( V_i \) and \( V_j \) are the voltages of \( i^{th} \) and \( j^{th} \) buses respectively and \( y_{ij} \) is the admittance of the line between \( i^{th} \) and \( j^{th} \) buses.

Reactive Power Flow

\[
Q_{ij} = \text{Imag}[V_i (V_i - V_j) y_{ij}^*]
\]

(3)

Here \( Q_{ij} \) is the reactive power flowing through the line connecting \( i^{th} \) and \( j^{th} \) buses.

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Real Power Loss

\[
\text{Loss} = \text{Re}\left\{\sum_{j \in \mathcal{N}} (V_{ss} - V_j) y_{ss,j} \right\} - \sum_{j=1}^{N} P_{Dj}
\]  

(4)

Where, \(V_{ss}\) and \(V_j\) in Equation 4 refers to the voltages at main substation and bus \(j\) respectively, \(y_{ss,j}\) refers to the line admittance between the main substation bus and bus \(j\), \(P_{Dj}\) refers to the real power load at bus \(j\), and \(N\) the number of buses in the RDS.

Voltage Deviation Index (VDI)

In order to quantify the extent of violation of limits imposed on voltages at buses in a RDS, the following Voltage Deviation Index (VDI) has been defined.

\[
VDI = \sqrt{\frac{\sum_{j=1}^{N} (V_{Li} - V_{L_{\text{LIM}}})^2}{N}}
\]  

(5)

Subject to \(V_{j_{\text{MIN}}} \leq V_j \leq V_{j_{\text{MAX}}} \quad j \in 1 \text{ to } N\)

Where, \(N_{VB}\) is the number of buses that violates the prescribed voltage limits and \(V_{L_{\text{LIM}}}\) is the upper limit of the \(i^{th}\) load bus voltage if there is upper limit violation or lower limit if there is a lower limit violation.

During reconfiguration, if the state of the system has voltage limit violations; the given solution must try and minimize the index VDI.

3. ALGORITHM DEVELOPMENT

A sample distribution system drawn below is taken here to illustrate the methodology [6].

![Fig. 1. Equivalent current injection based model of distribution network](image_url)

The distribution networks, the equivalent-current-injection-based model is more practical as shown in Figure 1. For bus \(i\), the complex load \(S_i\) is expressed by:

\[
S_i = (P_i + jQ_i) \quad i=1, \ldots, N
\]  

(6)

Corresponding equivalent current injection at the \(k^{th}\) iteration of solution is:

\[
I_i^k = I_i^k(V_i^k) + jI_i^k(V_i^k) = \left(\frac{P_i + jQ_i}{V_i^k}\right)^k
\]  

(7)

Where \(V_i^k\) and \(I_i^k\) are the bus voltage and equivalent current injection of bus \(i\) at the \(k^{th}\) iteration respectively. \(I_i^k\) and \(J_i^k\) are the real and imaginary parts of the equivalent current injection of bus \(i\) at the \(k^{th}\) iteration respectively.

Relationship Matrix Development

A sample power distribution system shown in Figure 1 is used as an example here. The power injection can be connected to the equivalent current injections by using Equation 7 and relationship between the bus current injections and branch current can be obtained by applying Kirchoff’s current law (KCL) to the distribution network. The branch currents can then be formulated as functions of equivalent current injections. For example the branch currents \(B_1\), \(B_2\) and \(B_3\) can be expressed by equivalent current injections as:

\[
B_1 = I_1 + I_1 + I_4 + I_5 + I_6
\]

\[
B_2 = I_4 + I_5
\]

\[
B_3 = I_6
\]

(8)

Therefore the relationship between the bus current injections and branch currents can be expressed as:

\[
\begin{bmatrix}
B_1 \\
B_2 \\
B_3
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6
\end{bmatrix}
\]

(9.1)

Above can be expressed in general form as:

\[
[B] = [BBCB][I]
\]  

(9.2)

The relationship between branch currents and bus voltages can be obtained as follows:

\[
V_2 = V_1 - B_1 Z_{12}
\]

(10.1)

\[
V_3 = V_2 - B_2 Z_{23}
\]

(10.2)

\[
V_4 = V_3 - B_3 Z_{34}
\]

(10.3)

Substituting Equations 10.1 and 10.2 into Equation 10.3, the Equation 10.3 can be written as:

\[
V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34}
\]

(11)

From Equation 11, it can be seen that the bus voltage can be expressed as a function of branch currents, line parameters and the substation voltage. Similar procedures can be performed on other buses; therefore the relationship between branch currents and bus voltages can be expressed as:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{bmatrix}
= 
\begin{bmatrix}
-Z_{12} & 0 & 0 & 0 & 0 & Z_{23} \\
Z_{12} & Z_{23} & 0 & 0 & 0 & Z_{34} \\
Z_{12} & Z_{23} & Z_{34} & 0 & 0 & Z_{45} \\
-Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 & Z_{36} \\
-Z_{12} & Z_{23} & Z_{34} & Z_{45} & Z_{36} & Z_{56}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4 \\
B_5 \\
B_6
\end{bmatrix}
\]

(12.1)

Above Equation 12.1 can be expressed in general form as:

\[
\text{DeltaV} = [\text{BCBV}][B]
\]  

(12.2)

Building Formulation Development

Observing Equation 9, a building algorithm for BIBC matrix can be developed as follows:

1) For a distribution system with \(m\) branch sections and \(n\) buses, the dimension of the BIBC matrix is \(m \times (n-1)\).

2) If a line section \(B_k\) is located between bus \(i\) and bus
j, copy the column of the \(i^{th}\) bus of the BIBC matrix to the column of the \(j^{th}\) bus and fill a +1 to the position of the \(k^{th}\) row and the \(j^{th}\) bus column.

3) Repeat procedure (2) until all line sections is included in the BIBC matrix. From Equation 12, a building algorithm for BCBV matrix can be developed as follows.

4) For distribution network with \(m\) branch sections and \(n\) buses, the dimension of BCBV matrix is \((n-1) \times m\).

5) If a line section \((B_k)\) is located between bus \(i\) and bus \(j\) copy the row of the \(i^{th}\) bus of BCBV matrix to the row of the \(j^{th}\) bus and fill the line impedances \((Z_{ij})\) to the positions of the \(j^{th}\) bus row and \(k^{th}\) column.

6) Repeat procedure 5 until all line sections is included in the BCBV matrix.

The algorithm can easily be expanded to a multiphase line sections or buses. For example, if the line section between bus \(i\) and bus \(j\) is a three phase line section, the corresponding branch current \(B_i\) will be a 3 x 1 vector and the +1 in the BIBC matrix will be a 3 x 3 identity matrix. Similarly if the line section between bus \(i\) and bus \(j\) is a three phase line section, the \(Z_{ij}\) in the BCBV matrix is a 3 x 3 impedance matrix.

**Solution Technique Developments**

The BIBC and BCBV matrices are developed based on the topological structure of distribution systems. The BIBC matrix represents the relationship between bus current injections and branch currents. The corresponding variations at branch currents, generated by the variations at bus current injection can be calculated directly by the BIBC matrix. The BCBV matrix represents the relationship between branch current and bus voltages. The corresponding variations at bus voltage, generated by the variations at branch currents can be calculated directly by the BCBV matrix. Combining Equations 9.2 and 12.2, the relationship between bus current injections and bus voltages can be expressed as:

\[
\Delta V = [BCBV][IBC][I] \\
\Delta V = [DLF][I] 
\]  
(13)

DLF is a multiplication matrix of BCBV and BIBC matrices and the solution for distribution load flow can be obtained by solving Equation 8 iteratively as:

\[
I^k_{it} = I^k_{i1} (V^k_i) + j I^k_{i2} (V^k_i) = \left(\frac{P^k_i + jQ^k_i}{V^k_k}\right)^* \\
[\Delta V]^{k+1} = [DLF][I]^k \\
[V^{k+1}] = [V^0] + [\Delta V]^{k+1} 
\]  
(14)

According to the research, the arithmetic operation and number for LU factorization is approximately proportional to \(N^3\). For a large value of \(N\), the LU factorization will occupy a large portion of the computational time. Therefore if the LU factorization can be avoided, the load flow method can save tremendous computational resource. From the solution technique described in this section, the LU decomposition and forward backward substitution of the Jacobian matrix are the Y admittance matrices, are no longer necessary. Only the DLF matrix is necessary in solving load flow problem. Therefore above discussed method can save considerable computation resources and this feature make the proposed method suitable for online operation.

**Flow Chart**

Figure 1 show a flowchart of the proposed method, which is based on two matrices that are combined to form a direct approach.

![Flow Chart](image)

**4. RESULTS AND ANALYSIS**

Distribution Load Flow (DLF) program has been tested on 33-bus RDS given in Figure 3. The load data, line details and the tie lines available for switching are given in appendix A. Substation voltage is 12.66 KV and base MVA has been taken as 10 MVA.

System has five tie lines. The two configurations are termed as Base Configuration and Optimal Configuration, respectively. Using DLF program voltages at the buses, real and reactive powers flowing through lines, real power loss and Voltage Deviation Index (VDI) were calculated for the two configurations.

Reduction in the real power loss, improved voltage deviation and increased bus voltages are the merits shown
by the method used. This can be understood by having a look on Tables 1 and 2.

**Fig. 3. A 33-Bus radial power distribution system**

**Table 1. Comparison of the two cases**

<table>
<thead>
<tr>
<th>Case</th>
<th>Loss(DLF/given in [21]) in KW</th>
<th>VDI(DLF/given in [21])</th>
<th>Worst Voltage(DLF/given in [21]) in p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>201.42/211</td>
<td>0.0174/0.02489</td>
<td>0.9143/0.9038</td>
</tr>
<tr>
<td>Optimal</td>
<td>158.24/178</td>
<td>0.0039/0.0041</td>
<td>0.9388/0.9378</td>
</tr>
</tbody>
</table>

**Table 2. Percentage comparison of the two cases**

<table>
<thead>
<tr>
<th>Case</th>
<th>%Loss Reduction</th>
<th>% VDI Improvement</th>
<th>% Increment in Worst Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>4.5</td>
<td>30</td>
<td>1.16</td>
</tr>
<tr>
<td>Optimal</td>
<td>11.1</td>
<td>4.8</td>
<td>0.106</td>
</tr>
</tbody>
</table>

**Voltage Comparison**

V1: Bus Voltages in per unit obtained from the DLF Program for base case.
V2: Bus Voltages in per unit obtained by using ETAP software.

**Comparison between the Bus Voltages**

Solutions to the voltages at the buses obtained show that at each bus, voltage in case DLF program is better than those in case of ETAP simulation results. Worst bus voltage in case of ETAP is 0.908 and that in case of DLF method it is 0.914. Also the best voltage is higher in case of DLF solutions. Once the voltages become higher, the losses are bound to be reduced. For the same load, power drawn in case of ETAP solutions is higher as compared to that obtained by DLF method. This only signifies the fact that losses in latter case have been reduced.

**Graphical Comparison of Base and Optimal Real and Reactive Power**

Figures 5 and 6 are showing the comparisons between real and reactive power respectively flowing through the lines in the two cases. In optimal case lesser real power is required because the loss has been decreased. This was the objective to be achieved through reconfiguration. This discussion is equally applicable to reactive power comparison.

**Table 3. Comparison of DLF and ETAP solutions**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V1(p.u)</th>
<th>V2(p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.997</td>
<td>0.996</td>
</tr>
<tr>
<td>3</td>
<td>0.992</td>
<td>0.984</td>
</tr>
<tr>
<td>4</td>
<td>0.985</td>
<td>0.975</td>
</tr>
<tr>
<td>5</td>
<td>0.977</td>
<td>0.968</td>
</tr>
<tr>
<td>6</td>
<td>0.959</td>
<td>0.951</td>
</tr>
<tr>
<td>7</td>
<td>0.956</td>
<td>0.947</td>
</tr>
<tr>
<td>8</td>
<td>0.942</td>
<td>0.934</td>
</tr>
<tr>
<td>9</td>
<td>0.936</td>
<td>0.929</td>
</tr>
<tr>
<td>10</td>
<td>0.93</td>
<td>0.923</td>
</tr>
<tr>
<td>11</td>
<td>0.929</td>
<td>0.922</td>
</tr>
<tr>
<td>12</td>
<td>0.928</td>
<td>0.921</td>
</tr>
<tr>
<td>13</td>
<td>0.922</td>
<td>0.915</td>
</tr>
<tr>
<td>14</td>
<td>0.919</td>
<td>0.913</td>
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<tr>
<td>15</td>
<td>0.918</td>
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<tr>
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<td>0.917</td>
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<td>0.915</td>
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<td>18</td>
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<td>20</td>
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<tr>
<td>23</td>
<td>0.989</td>
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<td>0.972</td>
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<td>25</td>
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</tr>
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<td>26</td>
<td>0.957</td>
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<td>27</td>
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<td>0.935</td>
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<td>31</td>
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<td>32</td>
<td>0.927</td>
<td>0.92</td>
</tr>
<tr>
<td>33</td>
<td>0.926</td>
<td>0.919</td>
</tr>
</tbody>
</table>
Fig. 4. Voltage comparison

Fig. 5. Real power comparison

Fig. 6. Reactive power comparison
Comparison of Base and Optimal Voltages

Figures 7 and 8 compares the results obtained for the two cases considered. It is concluded from the figure that voltages at the buses in case of optimal case is much better than that in the base case for majority of buses. Few buses have lower voltages (in case of optimal case) than that in base case. This is because of the fact that in the former case the structure of the network has been drastically changed as compared to that of later case.

Results obtained indicate that the approach to load flow solutions is much superior to the previous approaches such as used in [9] and ETAP software. For example even in base configuration the worst voltage is better than the worst voltage obtained through ETAP simulation. Also, voltages at the majority of buses are greater than those obtained by the other methods such as in [9] and ETAP simulation. Also, real and reactive powers drawn are lower for the same demand. This aspect leads the system to have lower losses and better VDI as shown by the results.

Results shown and compared in Tables 1 and 2. The voltage was improved by 4.5% and 11.1% in base case and optimal case, respectively. VDI was improved by 30% and 4.8% in base case and optimal case, respectively. Similarly, worst voltage was improved by 1.16% and 0.106% in base case and optimal case respectively.

5. CONCLUSION

In this paper, a new method to minimize energy losses reconfiguration in power distribution system using DLF program is presented. Two matrices that are developed from the topological characteristics of distribution systems are used to solve distribution problems. These two matrices are combined to form a direct approach for solving reconfiguration problems. The execution time is extremely smaller as compared to other recent methods reported in literature for radial distribution systems, such as fast decoupled and Gauss Implicit Z-matrix method. Here, we do not require to compute Z-matrix or jacobian.

Merit of used method is that it is very effective and solutions get conversed even in second equation there for the execution time of DLF program is quite small. This is the big advantage for distribution system where the load varies indiscriminately. Limitation of the program is that it can be used only for the radial distribution system and, not for meshed distribution systems and transmission systems.

REFERENCES


APPENDIX

Data for 33-bus radial distribution system used in test

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Sending bus</th>
<th>Receiving bus</th>
<th>Impedance (ohm) (R+X*i)</th>
<th>Load at receiving end bus (kW+kVAr*i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.0922+0.0477i</td>
<td>100+60i</td>
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<td>2</td>
<td>2</td>
<td>3</td>
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<tr>
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<td>0.366+0.1864i</td>
<td>120+80i</td>
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<td>4</td>
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<td>5</td>
<td>0.3811+0.1941i</td>
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<td>0.819+0.707i</td>
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