Commercialising Voltage Regulation in Nodal Electricity Spot Markets

Seon Gu Kim, Hugh Outhred, and Iain MacGill

School of Electrical Engineering and Telecommunications
The University of New South Wales, Sydney, NSW 2052
AUSTRALIA

ABSTRACT

Nodal electricity spot markets that incorporate AC network models usually permit voltage variation at the nodes within voltage constraints set by engineering considerations. These engineering constraints then influence the dispatch optimization. However, since market participants can have widely varying tolerances with respect to voltage variation, they therefore ‘value’ the voltage regulation services differently. Accordingly, it is more desirable that voltage regulation, i.e. commodity quality service, is approached by market based commercial method rather than centralised technical method in the market efficiency perspective. Moreover, this approach may permit more effective management of power system voltages during abnormal system operating conditions. In this paper, we explore a mechanism for commercialising voltage regulation using a ‘voltage value function (VVF)’ model. The conventional technical regulation and the VVF-based approach are numerically tested and compared in order to demonstrate the properties of the VVF model.

1. INTRODUCTION

Nodal electricity spot markets have now been implemented in a number of countries as part of electricity industry restructuring. These spot markets coordinate power system operation by having all industry participants submit regular ‘price-quantity’ bids to buy or sell electricity at their network locations. The market coordinator solves a dispatch optimisation based on these received bids subject to losses, flow limits and ‘quality of supply’ constraints arising from network operation.

One key parameter of ‘quality of supply’ is the variation in voltage at particular nodes in the power system. In standard nodal electricity spot markets, this is generally managed by setting an allowable range of voltage variation at the nodes – these become engineering constraints within the dispatch optimisation. The operation of some network equipment such as tap-changing transformers can be changed to assist voltage regulation. Also, market participants such as large generators may be required or rewarded (for example, through a centralised tendering process) by the market operator to provide voltage regulation services.

There are some important potential limitations to this ‘technical regulation’ of voltage variation within such markets. In practice, market participants can have widely varying tolerances to voltage variation. For example, an IT services company may require very strict voltage supply while some industrial processes might be fairly indifferent to large voltage swings. These users therefore ‘value’ voltage regulation services differently.

Also, network flow constraints are often a function of the allowed nodal voltage operating range. The intent of nodal pricing for electricity is to allow network losses and network flow constraints to influence the spot market solution, and nodal pricing is implemented by incorporating a network model in the spot market algorithm. The determination of network flow constraints can take on great commercial significance.
Given the different ‘value’ market participants may place on voltage quality of supply and the commercial implications of technical voltage constraints on spot market dispatch, this paper presents a possible market-based approach to voltage regulation. This approach has market participants submit their voltage regulation preferences to the spot market operator. The different values that participants place on voltage variation are incorporated into the dispatch process to optimise the tradeoffs in costs and benefits for tighter versus more relaxed voltage control at each network node. This dispatch also accounts for the effects of nodal voltages on network losses and flow constraints.

Commercialising voltage regulation services in this way may enhance the economic efficiency of industry operation while avoiding the potential conflicts between engineering and economic approaches to setting network flow constraints, and also reduces the need for separate voltage-related ancillary services by creating a unified economic model for both active energy and voltage quality.

Implementation of this concept requires the use of an AC network model in the spot market algorithm as well as the introduction of voltage value functions (VVF) by which market participants (generators, end-users and network service providers) express their willingness to accept off-nominal voltages [1].

The paper describes the model and illustrates its application to a five-node network. The outcomes are compared to a nodal market model that uses fixed nodal voltage limits. Issues associated with practical implementation are discussed.

2. NODEL ELECTRICITY SPOT MARKET MODEL

A nodal auction model (NAM) that incorporates network flow into the market solution algorithms is described in [1]. Assume that the network model in the auction process has N nodes and M participants. Participants located at various nodes submit offers to sell and bids to buy energy for a given time period (S_i, B_i) to the market operator. A bid or offer of participant i is represented by an energy quantity (q_i) and price per unit energy (p_i). Then, optimal dispatch of these bids and offers subject to network operating constraints is achieved with the following mathematic model for node k as shown in Eq. (1) [1].

\[
\text{Max} \quad \sum_{i=1}^{M} x_i \cdot p_i
\]

subject to:

\[
g_k (\sigma, x) = \sum_{b \in S_i \cup B_i} x_i \quad k = 1,2, \ldots, N
\]

\[
h_k (\sigma, x) \leq 0
\]

\[
0 \leq x_i \leq q_i \quad i \in \text{seller} \ (S_i)
\]

\[
-q_i \leq x_i \leq 0 \quad i \in \text{buyer} \ (B_i)
\]

where, 

- x_i = optimal dispatched generation and consumption associated with S_i and B_i,
- \(\sigma\) = the network state vector of voltage magnitude and voltage angle at all nodes,
- x = active power dispatch vector of participants,
- \(g_k (\sigma, x)\) = a vector value function expressing sum of power flow incident to node k,
- \(h_k (\sigma, x)\) = a vector valued function expressing constraints on the operation of the electricity industry such as line power flow,
\[ b_i = \text{participant’s bid data, and} \]
\[ q_i = \text{supplier’s capacity or consumer’s demand in bids} \]

The chosen network model used in the NAM determines how closely the market outcomes mirror the physical realities of network power flows. The function \( g \) \((\sigma, x)\) represents network ‘power flows’ [2]. Three general flow models are available - AC full load flow model, DC load flow model and TRANSPORT model [3]. Even though the simpler TRANSPORT and DC Load flow models have been used in spot market implementations to date, an AC load flow model is required if voltage regulation is to be addressed [4], and is hence used in the work presented here.

3. **VOLTAGE VALUE FUNCTION (VVF) MODEL**

Market participants can have widely varying tolerances to voltage variation, depending on the particular technical characteristics of their generation, network or end-use equipment. There will be some range of voltage variation that causes no problems at all.

While excessive voltage swings might permanently damage valuable equipment, some degree of variation beyond this acceptable range may, instead, temporarily inconvenience and cause financial losses to participants. For example, generators may have to restrict their active power output as the voltage at their connection node falls.

Some important aspects of this relationship between voltage variation and the impacts this has on participants might be modelled by having each participant express how the price they are willing to pay for energy (for consumers) or accept (for generators) changes with voltage variation at their node. Such price changes could compensate these participants for the losses or inconvenience of significant variation. [3, 5].

Let us assume that a consumer \( c \)’s nodal voltage region where there are no adverse impacts lies between \( V_{c_{\text{min}}} \) and \( V_{c_{\text{max}}} \), and their spot market bid price is \( p_c \) [$/MWh], representing the maximum allowable price they are prepared to pay for electric energy delivered at a voltage that lies within this boundary. Assume that the price elasticity of consumers with respect to voltage variation below their lower boundary and above their upper boundary is defined by \( \alpha_c \) and \( \beta_c \) respectively. If these elasticities are equal and piecewise linear, \( \alpha_c \) and \( \beta_c \) both equal.

\[ \alpha_c = \beta_c = \frac{\Delta P}{\Delta V}, \quad \alpha_c \leq 0, \beta_c \leq 0 \]

\[ (2) \]

![Fig. 1 Consumer’s Voltage Value Function](image)
Even though the curvature of price elasticity was assumed as linear change in the piecewise model above, the characteristic of curvature could have very various models with high order curves depended on the participants’ voltage preference.

Similarly, let us assume that a generator’s preferred node voltage boundary lies between \( V_{s_{\min}} \) and \( V_{s_{\max}} \), and its offer price is \( p_s \) [$/MWh], representing the minimum price at which it is willing to generate energy at a voltage within this voltage boundary. The price elasticity of a generator with respect to voltage variation below its lower boundary and above its upper boundary is defined \( \alpha_s \) and \( \beta_s \) respectively. If these elasticities are equal, \( \alpha_s \) and \( \beta_s \) both equal

\[
\alpha_s = \beta_s = \frac{\Delta P}{\Delta V}, \quad \alpha_s \geq 0, \beta_s \geq 0
\]  

(3)

![Fig. 2 Generator’s Voltage Value Function](image)

Based on the conceptual discussion above, the actual value function (VVF) model, which has 3rd order curvature and is sufficiently applied to nodal auction model, is suggested as shown in Eqs. (4) and (5). The bid or offer price \( p_i \) for a participant \( i \) located at node \( k \) can now be defined as:

\[
p_i = p_i^* \cdot FV_i(V_k)
\]  

(4)

where, \( p_i^* \) is the original bid or offer price for participant \( i \) and \( FV_i(V_k) \) is that participant’s Voltage Value Function (VVF).

\[
FV_i(V_k) = \begin{cases} 
1 + \alpha_i (V_{k_{\min}} - V_k)^3 & \text{if } V_k < V_{k_{\min}} \\
1 & \text{if } V_{k_{\min}} \leq V_k \leq V_{k_{\max}} \\
1 + \beta_i (V_k - V_{k_{\max}})^3 & \text{if } V_k > V_{k_{\min}}
\end{cases}
\]  

(5)

4. VOLTAGE QUALITY CO-OPTIMISED NODAL SPOT MARKET MODEL

Let us now consider a transmission system pie-equivalent model that has \( Y_k, Y_j \) (self-admittance), \( Y_{kj} \) (mutual admittance) and \( N \) nodes (\( k = 1, 2, 3, \ldots, N \)). A two-node example of this transmission system model is shown in Fig. 3.
The voltage quality co-optimised nodal spot market is mathematically modelled by incorporating the network related constraints and voltage value function into Nodal Auction Model (NAM) as follows [3, 6].

![Fig. 3 π-Equivalent Nodal Power Systems](image)

(1) **Objective function**

\[
\text{Max } \sum_{i=1}^{M} x_i \cdot p_i = \sum_{i=1}^{M} x_i \cdot p_i^* \cdot FV_i(V_k)
\]  

(6)

for participants,

(2) **Active power balance**

\[
\sum_{j=1}^{N} V_k V_j (G_{kj} \cos \theta_{kj} + B_{kj} \sin \theta_{kj}) + \sum_{b_j \in B_{kj}} x_i = 0
\]  

(7)

at nodes, \( k = 1, 2, \cdots, N \), \( j = 1, 2, \cdots, N \), \( G_{kj} \) and \( B_{kj} \) are the real and imaginary parts of mutual admittance (\( Y_{kj} \)) between node \( k \) and \( j \), \( \theta_{kj} \) is the voltage angle difference between node \( k \) and \( j \),

(3) **Reactive power balance**

\[
\sum_{j=1}^{N} V_k V_j (G_{kj} \sin \theta_{kj} - B_{kj} \cos \theta_{kj}) + y_k + \sum_{b_i \in B_i} \varphi_i \cdot x_i = 0
\]  

(8)

where, \( y_k \) is shunt admittance at node \( k \) and \( \varphi_i \) is power factor of load \( i \). For loads, reactive requirement is modelled by the affine function [7], which is proportional to active power as well as power factor (\( \varphi_i \)).

(4) **Reactive power generation**

\[
Q_k^{\text{min}} \leq Q_k \leq Q_k^{\text{max}}
\]  

(9)

where, \( Q_k^{\text{min}}, Q_k^{\text{max}} \) is the lower and upper limit of reactive power supply at node \( k \).
(5) Voltage constraint (only for technical voltage regulation)

\[ V_k^{\text{min}} \leq V_k \leq V_k^{\text{max}} \]  \hspace{5cm} (10)

where, \( V_k^{\text{min}}, V_k^{\text{max}} \) is the lower and upper limit of nodal voltage at node \( k \).

(6) Network flow constraint

\[ S_{kj}^{C_{\text{min}}} \leq S_{kj} \leq S_{kj}^{C_{\text{max}}} \]  \hspace{5cm} (11)

where, \( S_{kj}^{C_{\text{min}}}, S_{kj}^{C_{\text{max}}} \) is the lower and upper capacity of power flow for line between \( k \) and \( j \), and \( S_{kj} \) is apparent load flow from \( k \) to \( j \).

Since energy and voltage reservation prices are simultaneously incorporated and processed by the optimisation algorithm in this model, it is now effectively a ‘voltage quality co-optimised electricity market model’. Compared with a spot market model using technical voltage constraints, this model has some distinguishable characteristics:

- Market participants can simultaneously submit their preferences on energy prices as well as on voltage levels. Therefore, it is not necessary to organize separate commercial arrangements for voltage regulation services.
- The approach offers a possible solution for the ‘boundary issue’ between conventional spot energy markets and voltage-related ancillary service markets
- The magnitudes of the voltage tolerance parameters \((\alpha, \beta)\) provide numerical indicators for participants’ voltage quality preferences.

5. NUMERICAL ILLUSTRATION OF THE VVF MODEL

5.1 System Model: 5-Bus System

We simulate operation of the VVF model for the 5-bus system that was used in [8]. This system consists of two generator nodes and three load nodes as shown in Fig. 4. The parameters of this transmission network are given in Table 1, which is based on 100[MVA].

![5-Bus System Test Model](image)
Table 1 Transmission Parameter Data on 100 MVA base

<table>
<thead>
<tr>
<th>Line</th>
<th>From</th>
<th>To</th>
<th>R[p.u.]</th>
<th>X_L[p.u.]</th>
<th>X_C[p.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_1</td>
<td>N</td>
<td>S</td>
<td>0.02</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>L_2</td>
<td>N</td>
<td>L</td>
<td>0.08</td>
<td>0.24</td>
<td>0.05</td>
</tr>
<tr>
<td>L_3</td>
<td>S</td>
<td>L</td>
<td>0.06</td>
<td>0.18</td>
<td>0.04</td>
</tr>
<tr>
<td>L_4</td>
<td>S</td>
<td>M</td>
<td>0.06</td>
<td>0.18</td>
<td>0.04</td>
</tr>
<tr>
<td>L_5</td>
<td>S</td>
<td>E</td>
<td>0.04</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
<td>L_6</td>
<td>L</td>
<td>M</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>L_7</td>
<td>M</td>
<td>E</td>
<td>0.08</td>
<td>0.24</td>
<td>0.05</td>
</tr>
</tbody>
</table>

5.2 Market Data and Simulation

For this market-based simulation, we assume that each participant submits offers and bids to the market operator that reveal their willingness to buy or sell electric energy together with voltage value functions that express their willingness to accept off-nominal voltage at their node. Note that generators specify an allowable range of reactive power generation, whereas the reactive consumption of buyers is expressed by means of power factor.

The market participants submit bids and offers as shown in Table 2. Note that two bids or offers are submitted at each node. The market coordinator solves a dispatch optimisation based on these received bids subject to losses, flow limits and quality of supply constraints arising from the embedded AC load flow model of the network. To illustrate the properties of the VVF approach, five cases are simulated; one technical regulation case (where the voltage is held within a defined range of variation) and four VVF-based model cases with different voltage tolerance factors for the participants. This is summarised in Table 3.

Table 2 Offers and Bids Data (VVF Base Model - Case 2)

<table>
<thead>
<tr>
<th>Name</th>
<th>Bus</th>
<th>(V_{min})</th>
<th>(V_{max})</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(q)</th>
<th>(p)</th>
<th>(Q_{min})</th>
<th>(Q_{max})</th>
<th>P.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>N</td>
<td>0.95</td>
<td>1.05</td>
<td>40</td>
<td>40</td>
<td>0.75</td>
<td>20</td>
<td>-0.8</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>N2</td>
<td>N</td>
<td>0.95</td>
<td>1.05</td>
<td>40</td>
<td>40</td>
<td>0.70</td>
<td>40</td>
<td>-0.6</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>S1</td>
<td>S</td>
<td>0.95</td>
<td>1.05</td>
<td>40</td>
<td>40</td>
<td>0.40</td>
<td>30</td>
<td>-0.3</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>S2</td>
<td>S</td>
<td>0.95</td>
<td>1.05</td>
<td>40</td>
<td>40</td>
<td>0.20</td>
<td>60</td>
<td>-0.3</td>
<td>0.3</td>
<td>-</td>
</tr>
</tbody>
</table>

where, \(V_{min}\), \(V_{max}\), \(Q_{min}\), \(Q_{max}\), \(q\) are in [p.u.], \(p\) is in [$/MWh] .

Bids

<table>
<thead>
<tr>
<th>Name</th>
<th>Bus</th>
<th>(V_{min})</th>
<th>(V_{max})</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(q)</th>
<th>(p)</th>
<th>(Q_{min})</th>
<th>(Q_{max})</th>
<th>P.F</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>L</td>
<td>0.95</td>
<td>1.05</td>
<td>-40</td>
<td>-40</td>
<td>0.30</td>
<td>70</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>L2</td>
<td>L</td>
<td>0.95</td>
<td>1.05</td>
<td>-40</td>
<td>-40</td>
<td>0.15</td>
<td>60</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>M1</td>
<td>M</td>
<td>0.95</td>
<td>1.05</td>
<td>-40</td>
<td>-40</td>
<td>0.25</td>
<td>80</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>M2</td>
<td>M</td>
<td>0.95</td>
<td>1.05</td>
<td>-40</td>
<td>-40</td>
<td>0.15</td>
<td>70</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>E1</td>
<td>E</td>
<td>0.95</td>
<td>1.05</td>
<td>-40</td>
<td>-40</td>
<td>0.40</td>
<td>90</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>E2</td>
<td>E</td>
<td>0.95</td>
<td>1.05</td>
<td>-40</td>
<td>-40</td>
<td>0.20</td>
<td>80</td>
<td>-</td>
<td>-</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Table 3 Test Cases

<table>
<thead>
<tr>
<th>Model</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offer</td>
<td>$0.95 \leq V_k \leq 1.05$</td>
<td>$\alpha=40$</td>
<td>$\beta=40$</td>
<td>$\alpha=4000$</td>
<td>$\beta=4000$</td>
</tr>
<tr>
<td>Bid</td>
<td>$0.95 \leq V_k \leq 1.05$</td>
<td>$\alpha=-40$</td>
<td>$\beta=-40$</td>
<td>$\alpha=-4000$</td>
<td>$\beta=-4000$</td>
</tr>
</tbody>
</table>

5.3 Computation

The computation task for this auction process requires solution of an AC network flow model that includes non-linear equations. We perform this non-linear optimisation with sequential quadratic programming (SQP) implemented in MATLAB [9, 10].

The solution algorithm of SQP consists of three major steps as follows. The first is to find a search direction and design value $x$ for the current iteration through a Quadratic Programming (QP) subprogram. In this subprogram, the step size is determined by an appropriate line search direction. Secondly, the current iteration design value produced by the QP subprogram is used to test convergence to the optimal value. Finally, the Hessian matrix is updated using a quasi-Newton updating method for the next iteration if the solution of current iteration does not satisfy feasibility and optimality. Each step is described in detail in Fig. 5.

![Flowchart]

Fig. 5 NAM Optimisation Algorithm

The mathematical characteristics of our model are quite similar to classical optimal power flow solution approaches. Therefore, the main optimization scheme of the suggested VVF-based nodal auction algorithm is implemented with a well verified OPF package [11] that has been modified to implement a Nodal Auction Model.

5.4 Results and Analysis

At this early stage in our development of VVF, the main purpose of the simulations is to verify the robustness of our proposed VVF approach against the ‘technical regulation’ alternative. We
therefore compare the nodal voltages for four VVF cases against those voltages obtained with technical regulation.

As observed in Table 4 and shown in Fig. 6, even though the voltage profile in VVF model tends to be a little higher than technical method, both Case 3 and Case 5 that have very tight voltage preferences for the market offers result in nodal voltages very close to those for technical regulation. This suggests that the VVF methodology may be robust.

Table 4 Nodal Voltage Profile [p.u.]

<table>
<thead>
<tr>
<th>Model Case</th>
<th>N</th>
<th>S</th>
<th>L</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1.050</td>
<td>1.041</td>
<td>1.018</td>
<td>1.016</td>
<td>1.009</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.078 (2.67%)</td>
<td>1.068 (2.59%)</td>
<td>1.047 (2.85%)</td>
<td>1.045 (2.85%)</td>
<td>1.038 (2.87%)</td>
</tr>
<tr>
<td>Case 3</td>
<td>1.053 (0.29%)</td>
<td>1.044 (0.29%)</td>
<td>1.021 (0.29%)</td>
<td>1.020 (0.39%)</td>
<td>1.012 (0.30%)</td>
</tr>
<tr>
<td>Case 4</td>
<td>1.078 (2.67%)</td>
<td>1.068 (2.59%)</td>
<td>1.047 (2.85%)</td>
<td>1.045 (2.85%)</td>
<td>1.038 (2.87%)</td>
</tr>
<tr>
<td>Case 5</td>
<td>1.053 (0.29%)</td>
<td>1.044 (0.29%)</td>
<td>1.021 (0.29%)</td>
<td>1.020 (0.39%)</td>
<td>1.012 (0.30%)</td>
</tr>
</tbody>
</table>

Fig. 6 Nodal Voltages: (a) Distribution (b) Profile

6. CONCLUSIONS

Electricity industry restructuring is a complex process that typically involves the replacement of traditional engineering approaches to power system operation by more commercial, market-based arrangements. However, the physical realities of power system operation in areas including quality of supply restrict the scope of such commercial arrangements. Therefore, a somewhat uncomfortable mixture of commercial and engineering approaches is still evident in most electricity market implementations.

This paper has illustrated an approach for converting voltage regulation from conventional technical regulation to a market-based voltage value method. It has demonstrated that the inclusion of
7. REFERENCES


